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Equipment replacement: an integration view

Bradley Charles Meyer
Iowa State University

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Meyer, Bradley Charles, Ph.D.

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Equipment replacement:
An integration view

by

Bradley Charles Meyer

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Approved:

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Signature was redacted for privacy.
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For the Major Department

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For the Graduate College

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1989

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1 INTRODUCTION

In the last few years engineering economics has come under a great deal of criticism. The claim is that the commonly used models for making investment decisions are missing something. Many managers who are coming to the conclusion that they need to modernize their manufacturing methodologies to stay competitive turn to such models to aid their decision making process. But, unfortunately, when they make the estimates and plug the numbers into the economic replacement models, the answer too frequently comes up: "Don't invest". Being faced with a decision that their intuition tells them should be "yes", and an analysis technique that tells them "no", many conclude that the analysis technique must be flawed.

This is not to say that cash flow diagrams aren't valid, or that it is time to burn one's compound interest tables. Rather it seems that there are numerous factors relevant to replacement decisions that are commonly not considered when formulating economic analyses for replacement decisions.

Common lists of the benefits of modern manufacturing technology include such things as lower inventory levels, increased quality, decreased throughput time (from recognition of market desire to time when company can provide a product to meet that desire), and more flexibility in product characteristics. Such benefits, it is claimed, are not easily translatable into economic terms and thus are not easily

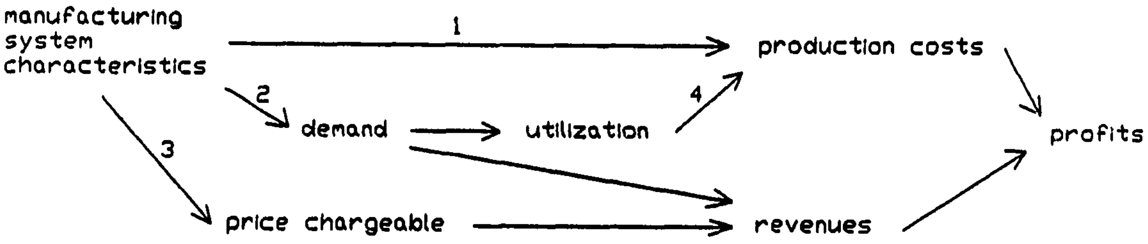

Figure 1.1: Direct and indirect effects of manufacturing methodology on production profitability

incorporated into economic models.

It is true that engineering economic approaches to equipment justification often consider only a subset of the economic impact of an expenditure. This is especially true when the replacement option is a machine which has different technological characteristics than the equipment currently in place.

The manufacturing methodology embodied in the collection of machines, devices, and people that make up the manufacturing system affects profitability in three ways. First, there is the direct effect of methodology on the costs of production. Second, there is an indirect effect through the market place that affects demand that affects utilization of the system that affects costs. Finally, there is the effect of the production methodology on product price chargeable.

Figure 1.1 depicts important relationships in modeling the economics of replacement. It is common for economic replacement models to consider only the machines-cost relationship (arrow 1). This research work develops methodology to integrate into the analysis techniques more of these relationships. In particular the relationships between manufacturing methodology and market demand (arrow 2) and between utilization and production costs (arrow 4) are addressed.

Chapter 2 provides a summary of the literature concerning the application of economic analysis techniques to justifying investments in modern manufacturing equipment and a summary of equipment replacement models. The third chapter outlines the objectives of the research.

The relationship between utilization and production costs and the resultant effect on replacement decisions is the subject of Chapter 4. Models are developed for a number of different modeling assumptions. The most general cases require the examination of a large number of possible equipment replacement sequences and dynamic programming algorithms are developed for their efficient solution.

The relationship between the manufacturing methodology and market demand requires that the replacement decision be considered in a system context, in contrast to the common 'single machine' approach. The manufacturing methodology is the composite of all the entities that make up the manufacturing system. In the most theoretical sense this includes all machines, material handling systems, employees, managers, and even designers. This concept is developed in Chapter 5 through a sample integrated system economics replacement model. Numerous observations are made about the model and about the general problems that will be encountered as such modeling methodology is used.

Chapter 6 summarizes the results and suggests topics of further research.

2 REVIEW OF RELEVANT LITERATURE

The literature review will be presented in two sections. First will be a discussion of traditional engineering economics modeling. This will focus on the shortcomings of traditional techniques to address the concerns of today's investment decisions. The second section will briefly detail the history of replacement analysis, including some innovative models that are being introduced today.

2.1 Weaknesses in Traditional Approaches to Justification

It is difficult to find an original way to begin a section discussing the shortcomings of traditional engineering economic analysis in justifying new manufacturing technology. So many articles have been written discussing this topic that most every kind of lead-in has been used at least once. At least five bibliographies on the subject have been published [Boothroyd 1984], [Canada 1986], [Fleisher 1982], [Hunter 1985], and [Sullivan 1984] and in 1986 the Institute of Industrial Engineers published a collection of significant articles [Meredith 1986].

First, it is important to identify what the "traditional" approach to justifying expenditures is. A sampling of articles reveals such descriptions as:

> "net present value, rate of return, payback, and other conventional economic analysis procedures" [Carrasco and Blank 1987, p. 211]

> "discounted cash flow as well as accounting-based methodologies" [Canada 1986, p. 137]
>
> "Engineering economy practitioners typically deal with tactical rather than strategic investment decisions" [Sullivan 1986, p.44]

The standard procedure used by an engineer in evaluating a potential capital investment would be the following:

1. Estimate the capital outlay required.
2. Estimate the year by year cash flows associated with this investment. These may include maintenance and energy costs, revenues generated, costs for tooling, taxes, and so forth.
3. Estimate the life of the investment and the salvage value at the end of its life.
4. Using these values compute some economic measure of merit that describes the investment. This is commonly one of three values: payback period, net present value, or rate of return.
5. Compare the investment with the status quo and with other potential investments. Select the best alternative(s).

The criticisms of this approach are many. They can be categorized into five major themes. (Interestingly enough, three of these are not a criticism of the method described above, but of the values, especially missing values, used in the computations.)

2.1.1 Failure to adequately consider indirect costs

As the labor component of today's manufacturing processes becomes smaller, the percent of manufacturing costs attributable to indirect costs becomes larger. "Insufficient detail on overhead spending and lack of models to explain overhead spending are industry wide problems" [Hunter 1985, p. 57]. Examples of such costs likely to be reduced with modern manufacturing technology are given in the literature and include inventory, rework and scrap, training, setup, and floor space [Meyer 1986], [Kaplan 1984]. While most engineering economics texts emphasize the importance of including such factors, they are commonly overlooked in practice.

2.1.2 Failure to adequately consider strategic factors

Strategic factors are perhaps the major compelling reasons for investing in robotics, CAD/CAM, group technology, AGVS, ASRS, and CIM technologies. Making such investments improves the long term competitiveness of a firm. Some of the strategic benefits resulting from modernization mentioned in the literature include shorter manufacturing through-put time, faster response to changing market demands, higher quality product, greater flexibility in manufacturing capabilities, improved worker morale, and the ability to deliver product at lower prices [Skinner 1984], [Bernard 1986], [Meredith and Suresh 1986]. Often, it is claimed, economic analysts ignore these factors feeling that these are the concern of upper level management. Yet, they are economic benefits, even if sometimes hard to quantify, and they should be included in economic analysis. Numerous authors make the point that estimates for these factors must be made since ignoring them is to assign them a value of zero, which is no more correct than an educated guess [Meyer 1986], [Hunter 1985].

2.1.3 Applying the wrong measure of merit

It is often stated in academic settings that the payback period criterion is a poor criterion to use for making capital budgeting decisions. Most engineering economists prefer present equivalent or rate of return criterion. Payback period owes its continued following to its ease of computation and intrinsic appeal to upper level managers who think in terms of turning over investment capital quickly. For some managers, their own personal rapid advancement within the company is more important than long term growth. This problem is described by Kaplan [1984].

2.1.4 Failure to consider the interaction of multiple investments within a firm [Leung and Tanchoco 1987], [Suresh and Meredith 1985].

The methodology of the traditional economic analysis procedure described above typically looks at investments in isolation from other investments. When other investments are included, for example, in the capital rationing scenario, usually the major interaction considered is the fact that the total investment capital is limited.

As the movement is made towards computer integrated manufacturing it is more difficult to examine any investment without considering the impact of that investment on the entire manufacturing environment. As one author stated, "though manufacturing is infinitely complex in all of its fine details, no single part can be treated in isolation.... Our own experience and the experience of others argues that a fractionalized approach just will not work!" [Lardner 1986, p. 73].

2.1.5 The use of a single number to measure the merit of an investment [Graham 1970]

The criticisms against this aspect of economic analysis focus on two aspects, intangible or nonmonetized factors and risk considerations. Since some of the strategy factors mentioned above and other factors (reputation for quality products for example) are hard to quantify, some authors recommend that the analysis leave these as nonmonetized for the decision maker to consider. The analyst provides these to management with simple rankings or descriptions. One author recommends the use of a structured project methodology in which such factors can be systematically evaluated [Bernard 1986].

A single value of a measure of merit conveys a limited amount of information about the economic ramifications of the investment. For example, if a manager is told that the prospective present worth of an investment is $20,000, the manager does not know such things as: "In the worst case, how much could we lose?" or "How much more than this amount is possible if we are lucky?" or "How many years will it take for us to realize this return on the investment?"

Risk analysis techniques have been included in engineering economic analysis for many years and are described in most textbooks. Smith [1987], and Riggs and West [1986] are two examples. There are techniques designed to avoid the condensation of the data into a single number and instead convey a range of values and their corresponding probabilities. Rather than choosing between the best of two simple values, alternatives are compared using statistical concepts such as stochastic dominance [Whitmore and Findlay 1978]. Although the mathematical tools for analyzing risk are available, often times such considerations are not included in economic analyses.

In summing up these criticisms of traditional methods of engineering economic analysis, it could be said that the literature points out a need in economic analysis to consider all the cost factors involved, to consider the systems effect of investment opportunities, and to present the data to the decision makers in a way that conveys as much information as possible.

2.2 Suggestions for Improvements

The literature also contains many prescriptions for improving the ability of economic analysis to be used in justifying modern technologies. Some of these techniques are small improvements to the current procedures while others are radical revisions of current practices. These will be briefly discussed starting with the smaller, incremental changes and moving to the more extensive revisions of analysis methodology. The interested reader is referred to Sullivan [1986] and Meredith and Suresh [1986] for other categorizations of techniques.

2.2.1 Better use of the current methodology by a careful assessment of indirect and intangible costs

Some articles propounding this as a solution give concrete examples of how to quantify commonly overlooked costs. Quantitative treatments include discussions of operating inferiority costs [Lowe 1987], robotics costs [Meyer 1986] and FMS costs [Klahorst 1983]. Articles that describe such costs qualitatively include [Kaplan 1984], [Blank 1985], [Phillips 1983], and [Primrose and Leonard 1986]. Meyer [1986, p. 120], states that, "management seems willing to relax the financial justification requirements for that 'first installation'. However, after the honeymoon, the realities of financially justifying the future systems require us to put some 'hard numbers' on all those wonderful benefits." This willingness to relax the requirements for first installations can be a vital opportunity since the benefits of the first installation can be tracked after the fact providing a basis for estimates of benefits of future investments.

2.2.2 Weighted multi-factor decision techniques

Factor analysis techniques have been used for many years. These are techniques where intangible values are assessed by assigning a numeric value that quantifies the benefit and the importance of that benefit to the decision maker. The interested reader is referred to Sullivan [1984], Frazelle [1985], Morris [1977], and Riggs and West [1986].

2.2.3 Operations research methodology

To handle the complex interactions of investment decisions in today's integrated manufacturing setting, many of the techniques of operations research can be used. A dynamic programming approach to machine replacement has been developed for considering the optimal pattern of replacements over a given time horizon [Oakford, Lohmann, and Salazar 1984]. While this model did not directly address multiple concurrent investments, it could be extended to include them. Linear programming is also suggested in selecting optimal machine investments [Srinivasan and Kim 1987]. A model that includes risk considerations within the framework of replacing machines in a flexible manufacturing cell is described by Leung and Tanchoco [1987].

Numerous authors suggest that simulation is a feasible way to access the effects of the complex interactions of integrated manufacturing [Carrasco and Blank 1987], [Blank 1985]. Blank, [1985, p. 237] comments that simulation languages will have to advance to produce results and allow simulated decisions "on a cost basis rather than bases such as queue length, waiting time, and priorities that may not use cost-sensitive weighting functions."

2.2.4 Models that encourage better cost estimation

Two articles recommend systems that involve iterative approaches to the investment decision making process. The cost estimates used in the approaches improve with each iteration. In one model, a cost tracking information system is set up within the plant. As the investment alternative moves from the planning stages to the implementation stages costs are captured and the cost/benefit criterion can be refined to reflect reality more closely [Carrasco and Blank 1987]. In another model, sensitivity analysis is performed to determine which factors will have the most impact on the total value of the investment. As these factors are revealed, more energy can go into estimating their actual worth [Miltenburg and Krinsky 1987].

2.2.5 Expert systems

Expert system applications in engineering economics consist of two types. Most useful to researchers are those well-documented, research-oriented models. Two such examples are [LeClaire and Sullivan 1985] and [O'Leary 1987]. The first of these two, entitled XVENTURE, was created using an expert system shell. The expert whose knowledge was captured was William Sullivan, a professor of Industrial

Engineering at the University of Tennessee. The system was a small one, consisting of six questions with anywhere from two to four possible responses for each question. This made a total of 648 combinations of responses possible. A most useful feature about XVENTURE is that the knowledge on which it operates focuses on the intangible factors in the investment decision. One question asked is whether the present equivalent worth of the investment opportunity is greater or less than zero. The remaining five questions ask such things as what is the investment's impact on capacity and quality, and how well does the technical plan involving this investment match the corporation's overall business strategy.

The other type of expert system in economic analysis is the commercial type. Of course, the knowledge representation scheme used in this kind of system and the reasoning techniques are not available for public examination.

2.2.6 Revise accounting systems

Perhaps the root of the problem of computing indirect costs is the continued use of cost accounting systems that were designed years ago when direct labor and direct materials were the major cost components in manufacturing. Often times overhead costs are allocated to products by merely adding a percentage to the direct labor cost. Since these accounting methodologies are used within the plant to determine manufacturing cost, it might seem reasonable that cost estimates prepared for capital expenditure alternatives use the same methodology. However, this will almost always underestimate the benefits of an investment in modern manufacturing techniques as they are well known for their ability to reduce indirect expenses. To solve this dilemma it is recommended by some not merely to compute cost studies differently, but to ultimately do a major overhaul of accounting systems used in manufacturing [Kaplan 1984].

This ends the review of current perspectives and ideas concerning the use of engineering economic analysis in justifying new manufacturing technology. The criticisms of traditional methods have been summarized and recommended alterations or replacements for those models have been described.

2.3 Replacement Theory

The above section described the state of economic analysis in general. Although most of the discussion focused around the problem of justifying new equipment–which implies replacement–little was said about the topic of replacement theory which has developed within the realms of engineering economy. A brief history of replacement theory will now be given with special attention to two issues. The first of these is the modeling of equipment degradation–the main reason for replacement. The second is the modeling of interactions of machines within a production system. This is a topic which has only recently been addressed.

2.3.1 Replacement and degradation functions

The major issue of economic analysis for equipment replacement is the decline in value of equipment over time. Indeed, the articles by Taylor [1923] and Hotelling [1925], that are often spoken of as being the starting point of replacement theory were actually articles on valuation–theories of how to determine depreciation. In these two articles, the decline in value of a machine stemmed from two time-dependent functions: that of operating costs, which increased with age, and that of salvage value, which decreased with age or perhaps remained constant. Taylor did not discuss these functions in depth, but Hotelling noted that operating costs in reality are dependent on both age and use [1925, p. 352]. However, most of his theory is developed using the assumption of constant utilization of the equipment, which makes operating costs practically a function of time only.

Preinreich [1940] considered the question of optimal replacement timing, divorcing himself from the depreciation viewpoint of Taylor and Hotelling. He modified their models and set the replacement decision in its rightful context as one decision in a sequential chain of decisions. His degradation functions were again time dependent only, based on an implicit assumption of constant utilization over time.

A significant advance in the modeling of degradation was made by Terborgh [1949], [1958] who elaborated the distinction between deterioration and obsolescence. Deterioration, he said, it a characteristic of the machine itself and the aging process. Obsolescence is another phenomenon that could encourage replacement, but appears not as a characteristic of a machine itself, but as a characteristic of the relative capabilities of new machines, called challengers, to those of the currently owned machine, called the defender.

Terborgh discussed the fact that operating costs increase as a function of use as well as of age. His book includes graphs showing cost as a function of use for numerous types of equipment on which data were available [1949, pp. 70-71]. While Terborgh realized that deterioration was a function of use, he did not feel that adequate data existed to make use of this observation, thus his well known MAPI method treated operating costs as increasing linearly as a function of time only. Salvage value was assumed to be negligible. Obsolescence was modeled through the existence of competing machines whose first year operating costs decreased with the year of their availability.

Since Terborgh, models in the literature have incorporated a variety of degradation functions. For example, Eilon, King, and Hutchinson [1966] employ monotonically decreasing salvage values, Grinyer [1973] uses negative exponential functions for the decline in first year operating costs of challengers, and Tanchoco and Leung [1987] use monotonically increasing operating cost coefficients. In all cases, the functions are time dependent, not use dependent.

When the degradation functions are modeled with the least amount of restrictions, the optimum replacement timing problem requires the comparison of a large number of possible decision sequences. Dynamic programming has emerged as the dominant technique to limit the computational burden of finding an optimal replacement strategy [Meyer 1981]. Bellman [1955] presented the first application of dynamic programming to replacement. Dreyfus [1957] expanded this two years later. Since then, numerous authors have presented dynamic programming formulations in replacement theory. Among them are Oakford, Lohmann, and Salazar[1984], Hastings [1973], and Leung and Tanchoco [1987].

The contribution of replacement analysis to production economics is its recognition of the phenomenon of the change in relative capabilities of equipment over time. To date this degradation has been modeled strictly as a function of age. While it is commonly acknowledged that the deterioration of equipment is also dependent on its utilization, this has not yet been addressed explicitly in replacement models.

2.3.2 Replacement and system interactions

While degradation is well known in replacement theory, integration is not. Few replacement models incorporate the fact that equipment items operate as components of manufacturing systems. The characteristics of each equipment item affect the costs and benefits of the other machines in the system. The replacement of a machine is not an isolatable event, thus replacement decisions should be based on consequences to the overall system. This is especially true in light of today's increasingly integrated manufacturing environment. Blank [1985] details the need of economic analysis models to incorporate this "integration view".

While the bulk of replacement analysis does not regard the issue of machine interactions, as early as 1968, replacement models including such facets have appeared in the literature [Hansmann 1968], [Ray 1971]. Earlier than this, Smith [1961] comments on multi-machine issues but leaves their development to the reader. A promising approach has recently been described in an article by Leung and Tanchoco [1987]. Their model includes the effects of replacement decisions on such system-oriented characteristics as the assignment of parts to machines, material handling costs, and utilization of manufacturing resources. Their model as presented, however, was simplified to include only a single time period which "rules out the effects of such time-dependent factors as obsolescence, deterioration, and future revenue/parts changes" [1987, p. 97]. They indicate that a more advanced model including such factors is soon to be published.

2.4 Summary

The survey of literature on economic analysis methods in general showed that to adequately address the needs of decision makers in today's manufacturing environment models must (1) be thorough in the modeling of tangible cash flows affected by the decision including those cash flows that result from the interactions of the components in the system, (2) seek to incorporate strategic, intangible factors along with the tangible cash flow values, and (3) provide to management more than a simple, single-valued measure to describe each alternative.

The survey of replacement models showed that while there is admission that equipment costs and benefits are dependent on the utilization of the equipment, models are generally formulated without explicitly incorporating it. Also, replacement models generally treat machines in isolation of the manufacturing system of which they are a part.

3 RESEARCH OBJECTIVES

The shortcoming of economic analysis models, in the context of equipment replacement in an environment characterized by rapid technological progress, is that the models allow too much to be left out. The concept of discounted cash flows is sound, as long as you define "cash flows" broadly enough to include all relevant financial impacts, direct or indirect. The models provide all the framework necessary to correctly evaluate replacement decisions, only if the analyst is conscientious enough to include all factors.

The objective in this research is to devise a more complete approach to the modeling of replacement economics that brings to the surface many of the factors that are usually considered to be "intangible" or "indirect." This forces the analyst to grapple with these factors and does not allow them to be ignored. Such intangibles and indirect costs can be incorporated at the modeling stage in a way that makes their quantification explicit and computable; that is, able to be found from shop floor data, through market research, from historical data, or some other "tangible" means.

The approach taken is an integrated approach. The capital costs, operating costs, and revenues pertaining to an entire manufacturing system made up of machines, humans, computers, and material handling systems, are combined in a model to analyze the economic effects of possible replacement scenarios.

There are two key concepts in the modeling approach. The first of these is utilization based operating costs and operating cost degradation functions. When technological change is a factor in replacement, there are two reasons why utilization based costs should be used. A change in an organization's manufacturing methods will be likely to effect the reputation of the product and firm through changes in quality, timely delivery, and so forth. Reputation, of course, is a major factor in determining demand for the product and the price that can be charged. Capital and operating costs are quite sensitive to the demand especially when expensive machinery is used. Capital costs are recovered through products produced by the equipment. Operating costs, direct and indirect, are dependent on the number of items produced.

The second key concept of this approach is the system orientation. Replacement decisions are evaluated with respect to their impact on the total manufacturing environment. The system approach as presented here calls for the detailed description of the interactions of the various components of the manufacturing system. By including machines, line workers, managers, material handling systems, and inventory storage areas, the tracking of indirect costs becomes an explicit rather than implicit task. Again, this prevents the analyst from ignoring these factors.

The strategy taken is to first extend a typical replacement model by adding the relationships between demand, utilization, and production costs. Thus, in Chapter 4, the concept of utilization based costs and degradation functions are developed within the context of single item equipment replacement models. The effects of the production characteristics on the demand are not addressed in this section. Rather demand is treated as independent of the equipment.

In the fifth chapter, the concepts developed in Chapter 4 are expanded to a manufacturing system model that incorporates system component interactions. Also this section treats demand as based on the production system characteristics. To simplify the model, the only characteristic of the system that affects demand is "product quality". The modeling approach could be extended to cover other characteristics such as throughput time.

The models presented are accompanied by numerical examples of their use. In these examples, numerous functions expressing the relationships described above are used. While these functions are chosen as reasonably likely relationships, there is no discussion of how the functions are derived. It is assumed that such relationships can be found from historical data or estimated from market research or equipment specification documentation. The issue of formulating such functions could easily be the topic of an entire research program. The objective of this research is to explore the integration of such relationships, given that they can be found.

4 UTILIZATION

4.1 Introduction

As stated in the literature review, economic models of replacement rarely include utilization explicitly. This chapter develops methodology for modeling utilization and for solving replacement questions in a context where only one machine is considered, or two machines performing the same process.

4.2 Modeling Equipment Cash Flows Incorporating Utilization

The typical cash flow items in replacement analysis are first cost, operating expenses, salvage value, and in some cases, revenues. The last three of these can be modeled as functions of utilization. The cumulative utilization will be denoted by cu. Usually this will be expressed as $cu(N)$ meaning the cumulative utilization at the end of year N. Cumulative utilization can be in terms of cumulative years, as will be the case in this chapter, or as cumulative hours, as will be the case in Chapter 5. When utilization is a constant, U, each year (U is between 0 and 1), then equation 4.1 holds.

$$cu(N) = NU \tag{4.1}$$

Salvage value is a function of age and utilization. $V(N, cu(N))$ is the salvage value of a machine that has been used for N years accumulating $cu(N)$ years of utilization. It would be reasonable that the salvage value be monotonically decreasing as N and cu grow larger. The value could very well become negative.

The operating expenses are modeled as a combination of two cost functions. First are those costs which are dependent only on time, such as depreciation tax effects, insurance, and property taxes. These will be called the age related operating costs, or $aroc(N)$. This function is defined as the cost per year to operate the equipment in the Nth year of operation. The use related operating costs, $uroc(cu)$, are those costs that are dependent on the amount of utilization, such as energy costs, maintenance, and repair. Let $uroc(cu)$ equal the cost per unit time to use the equipment when it has been used for a cumulative number of cu years.

For use with discrete cash flow analysis another function can be defined that cumulates the use related operating costs for a year N. Let $UROC(N)$ be the use related operating costs incurred in year N. This is the integral of $uroc(cu)$ from the value of cu at the beginning of year N to the value of cu at the end of year N. This is shown in equation 4.2.

$$UROC(N) = \int_{cu(N-1)}^{cu(N)} uroc(cu)dcu \tag{4.2}$$

It is generally true that the operating costs of a machine increase with time and with cumulative use. For most types of equipment this is true. For machines that make use of computer programs an opposite result might occur. Computer programs often have the characteristic of improving with age, because bugs will be found and corrected and the maintenance costs for the program will decrease. However, if the program is being used in a changing environment and must be updated to reflect changes in its use, then the changes will introduce new bugs and the cost decline may not be seen. We can summarize by saying that in general *aroc* and *UROC* are monotonically increasing functions of N and *cu*. For certain categories of equipment such as computer equipment, however *UROC* might actually decrease with increasing cumulative utilization.

Operating revenues are a function of utilization if the assumption is made that the more product produced, the more product sold. However, there is a difficulty in assigning revenue to any one machine in a manufacturing system where a part is produced by means of operations on many different machines. In this chapter, either fixed revenue will be assumed, or costs will be figured on a per part basis.

The functions V, *aroc*, and *UROC* are projections of the future. They could be calculated from extrapolations of historical data or from operating characteristic data of a new machine. A useful area of research not addressed in this treatise would be to determine how well these functions can be predicted and from what kinds of data.

Let B represent the first cost of a machine and i the interest rate. Given this and the variables defined above, the annual equivalent costs for an equipment item are given by equation 4.3.

$$AEC = \left(B + \sum_{n=1}^{N} (aroc(n) + UROC(n))\,(P/F)_n^i\right)(A/P)_N^i - V(N, cu(N))(A/F)_N^i \tag{4.3}$$

4.3 Cases and Assumptions

To demonstrate some of the effects that utilization has on replacement decisions four example analyses will be presented. The methodology for solving a replacement

Table 4.1: Examples presented and assumptions made

		constant service	varying service
1 machine	like-for-like replacement	example 1 (4.4)	↑
	different replacement	⟵	example 3 (4.6)
more than 1 machine	like-for-like replacement	example 2 (4.5)	↑
	different replacement	⟵	example 4 (4.7)

problem depends on the assumptions of the problem definition. In the examples that follow, the questions that differentiate between methods are:

1. How many machines are required to supply the desired demand? (1 or more than one)
2. Do replacement machines have equal cost characteristics?
3. Is the demand for the services of the machine constant or varying over time.

Table 4.1 shows the combinations of assumptions that are relevant to each example.

4.4 One Machine, Like-for-like Replacement, Constant Service Need

With the given assumptions, the replacement problem becomes one of determining the optimal number of years to keep a machine before replacing it with an identical one. This is found by simply finding the n value that minimizes the AEC value as given in equation 4.3. An example problem is shown below.

$$B = 30,000$$
$$V(n, cu(n)) = \left(\frac{.5B}{(1+i)^{2n}}\right) + \left(\frac{.5B}{(1+i)^{2cu(n)}}\right)$$
$$aroc(n) = 300(n-1)$$
$$UROC(n) = 1200cu(n)$$
$$cu(n) = nU$$
$$i = .20$$

In this example case, V is a monotonically decreasing function. $UROC$ and $aroc$ are monotonically increasing. The annual equivalent cost of owning this item for a life of N years is given below.

$$AEC(N) = 30,000(A/P)_N^2 + (300 + 1200U)\,(A/G)_N^2$$
$$- \left(\left(\frac{15,000}{(1.2)^{2N}}\right) + \left(\frac{15,000}{(1.2)^{2NU}}\right)\right)(A/F)_N^2$$

For $U = 1$ (100% utilization of the equipment) the AEC values for the first 10 values of N are as shown in Table 4.2. From these values it can be seen that the economic life is 7 years with an AEC of \$11,577. This is shown graphically in Figure 4.1.

Varying the utilization from 100% to 50% produces Table 4.3 showing the economic life, the AEC and the normalized cost per "unit" of production (using 100% as 1). Figures 4.2 and 4.3 show graphically the minimum AEC and the normalized cost per unit as a function of the utilization rate.

Two observations can be made from this example. First, as expected, the annual equivalent costs are lower as utilization drops. Second, the cost of the

Table 4.2: *AEC* with 100% utilization

N	AEC
1	15167
2	13742
3	12800
4	12200
5	11841
6	11651
7	11577
8	11583
9	11643
10	11736
11	11850
12	11975

Table 4.3: Normalized cost of machine use under varying levels of machine utilization

U	economic life	AEC	normalized cost/unit
100%	7	11577	1.000
95%	8	11421	1.038
90%	8	11257	1.080
85%	8	11093	1.127
80%	8	10926	1.180
75%	9	10757	1.239
70%	9	10576	1.305
65%	9	10393	1.381
60%	9	10208	1.470
55%	10	10012	1.573
50%	10	9814	1.695

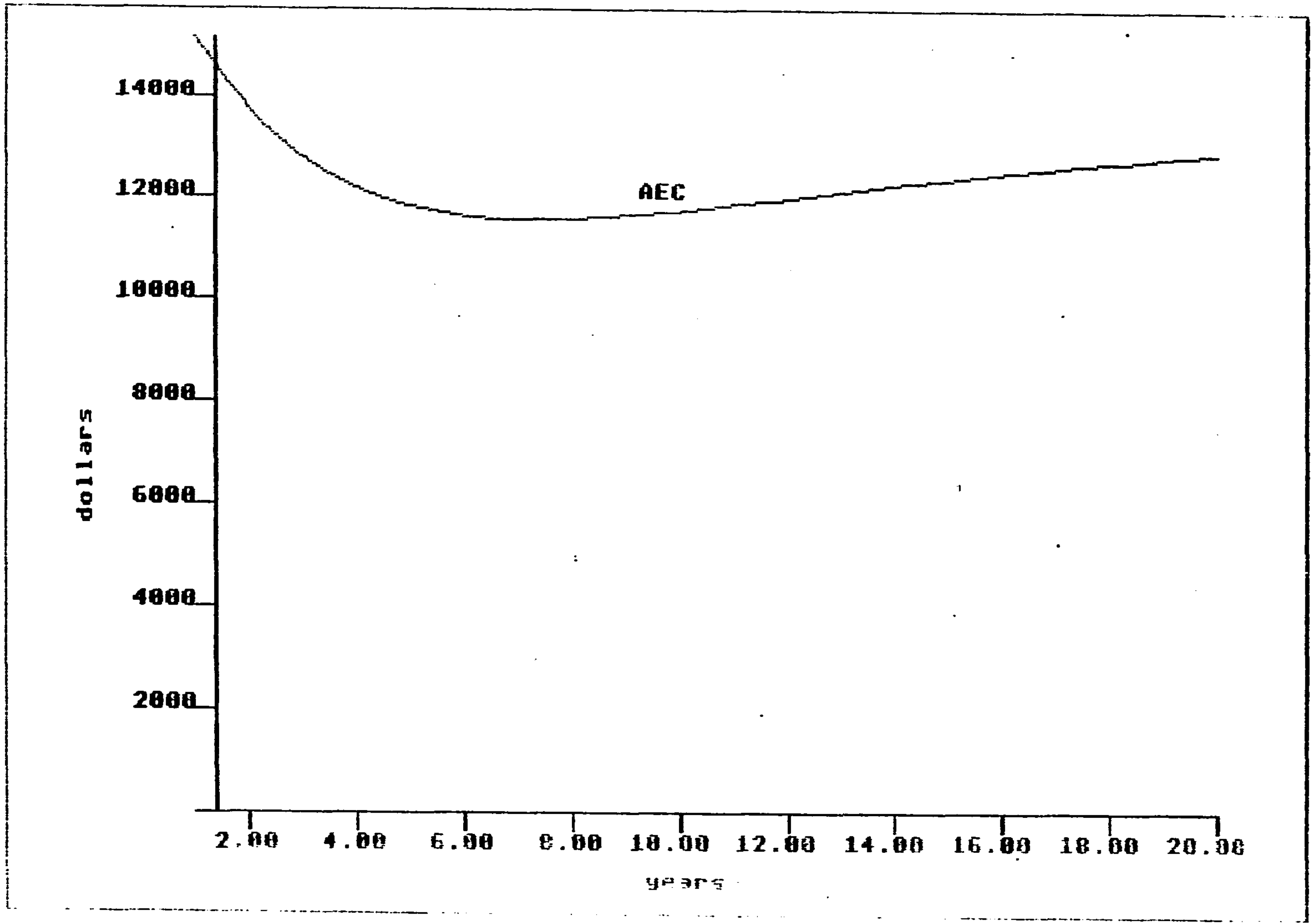

Figure 4.1: AEC as a function of N

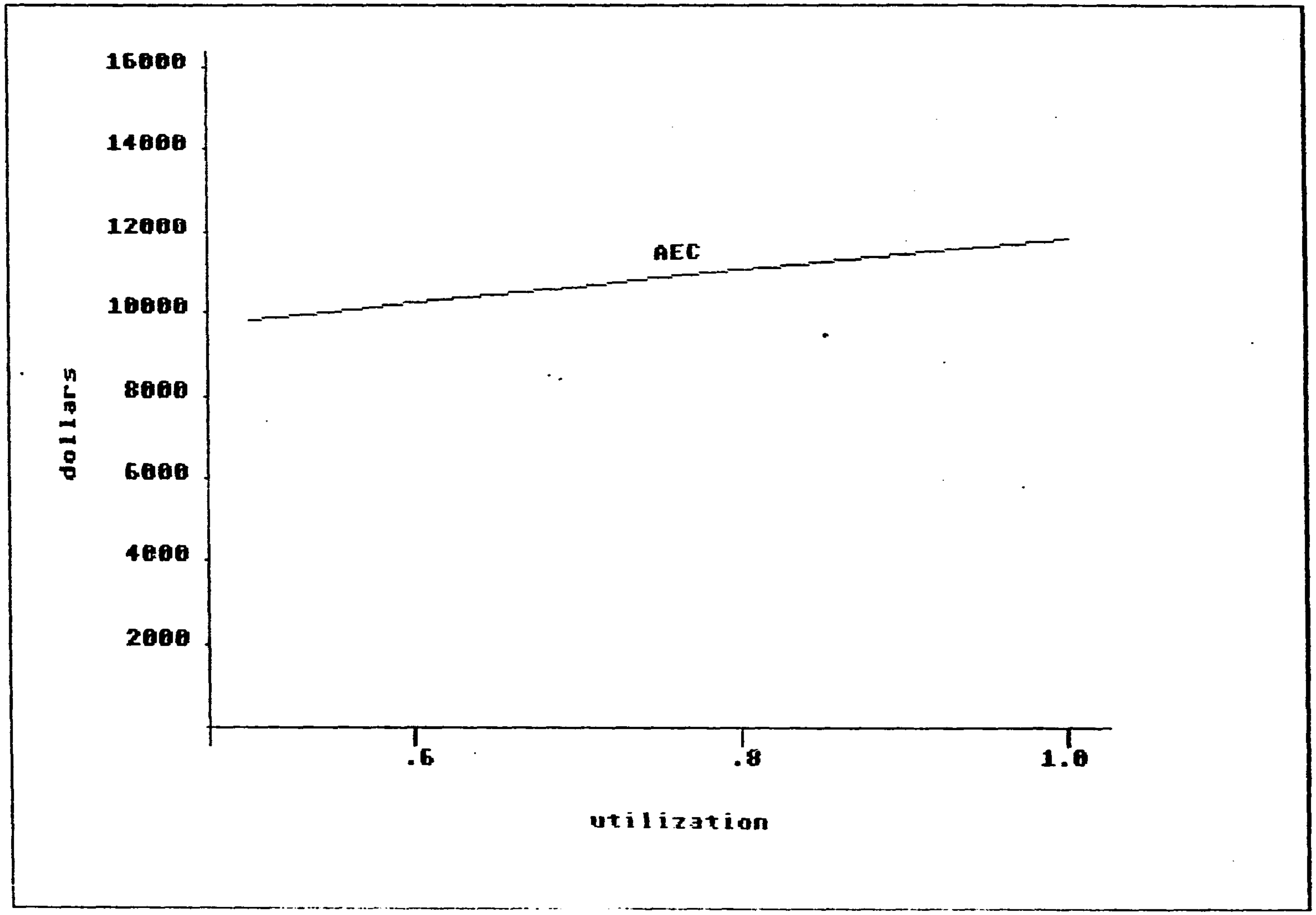

Figure 4.2: Minimum AEC as a function of utilization

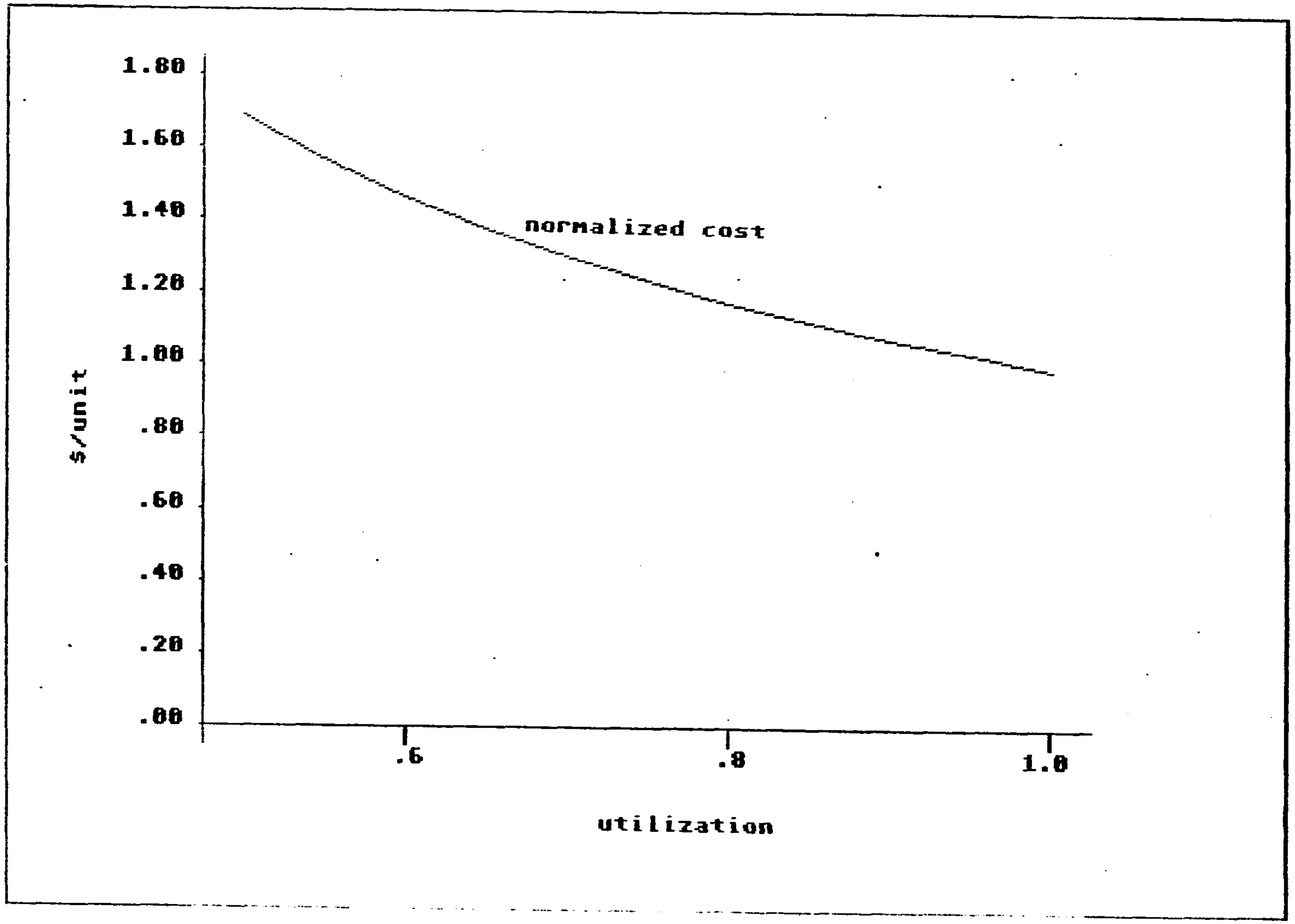

Figure 4.3: Normalized cost per unit as a function of utilization

machine, in terms of its per unit impact on the cost of the product produced grows significantly as utilization drops.

Since this is just one sample problem which does not necessarily lead to generalizations, an informal sensitivity is presented to show for various types of salvage value and operating expense functions what effect changes in utilization have on economic life and minimum AEC. Figure 4.4 shows a typical economic life curve. The costs are composed of two major contributors, capital cost and operating cost. The economic life curve can be divided into three sections. The first section manifests mainly the capital cost as the new equipment is quickly losing value. Following this, section two is fairly flat as the capital cost and operating cost gradients almost cancel each other out. Finally, in the third section of the curve, the operating costs become large enough proportionally so that they begin to dominate the curve.

In Figure 4.5 is shown a set of economic life curves for various values of utilization. There is a slow shifting of the economic life as utilization changes. In the first few years the salvage value, and thus utilization, has a significant effect on the first part of the curve. The use related operating costs can influence the latter part of the curve.

When utilization is less than 100% the decline in salvage value is slower and the gradient costs increase more slowly. Thus the economic life curves become flatter as the utilization goes down.

The sensitivity analysis considers a number of likely functional relationships for V and $UROC$ with utilization. In all, the 243 possible combinations of 3 salvage value relationships, 9 *aroc* functions and 9 $UROC$ functions are examined at 3 utilization levels. In the results the economic life length is only sensitive to utilization

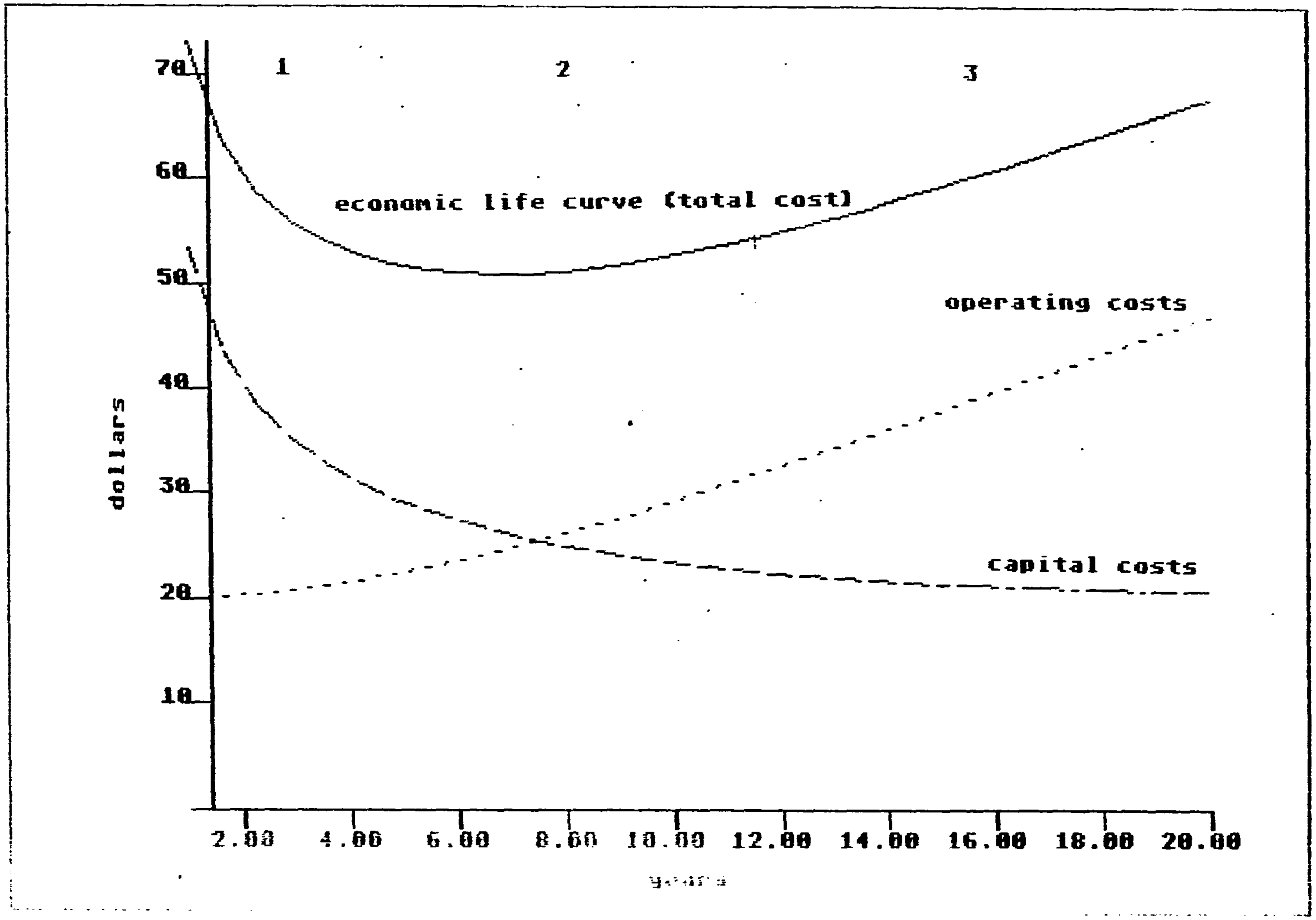

Figure 4.4: A typical economic life curve

in the cases where the economic life curve has a long flat region. (That is, in cases where the exact determination of economic life is not critical.) In all of the cases where the economic life changes more than one year as utilization varies from 100% to 50%, the optimum *AEC* value is in a range that does not vary more than 5% in a spread of 17 or more years. The sensitivity of economic life to changes in utilization is shown in Figures 4.6 through 4.9. Figures 4.6 and 4.8 summarize for the 243 cases how many years different the economic life is when utilization drops to 75% and 50% respectively. Figure 4.7 and 4.9 show for those same values of economic life shift, how wide the 5% and 10% windows are. The "windows" include all years of life for which the *AEC* value is less than or equal to 5% or 10% more than the minimum.

While economic life may not be very sensitive to changing levels of utilization, the cost per part of product produced is, as expected, quite sensitive to changing utilization levels. Figure 4.3 previously discussed is representative of the relationship between cost per part and utilization.

The functions used in the sensitivity analysis along with sample results are shown in Appendix A.

4.5 More Than One Machine, Like-for-like Replacement, Constant Service Need

Consider a case where two machines are required to meet the demand for a single product. The two machines have different capacities and different cost parameters associated with their use. It will be assumed that the like-for-like replacement assumption applies and that the demand for the product will not change over

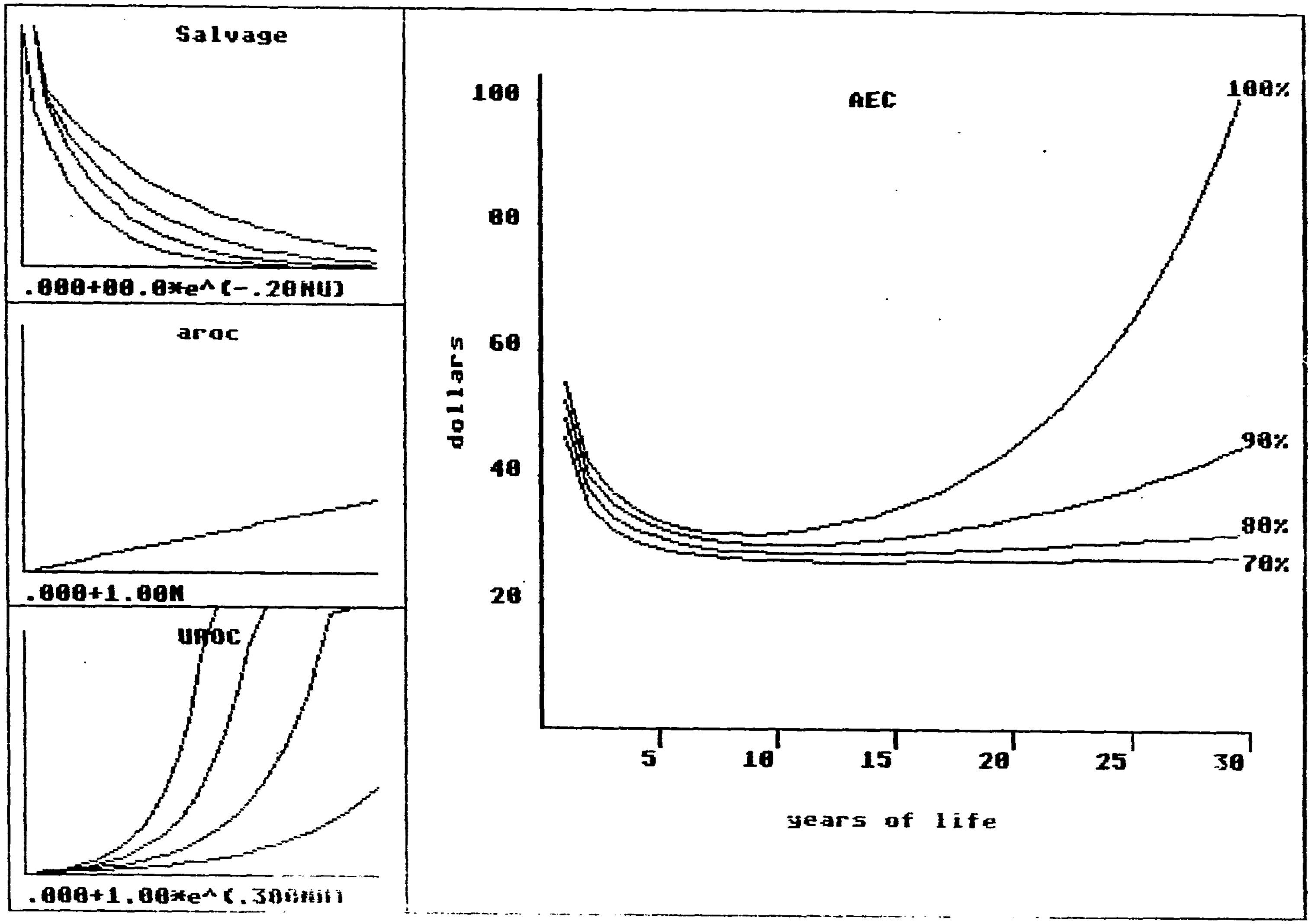

Figure 4.5: Economic life curves for various values of utilization

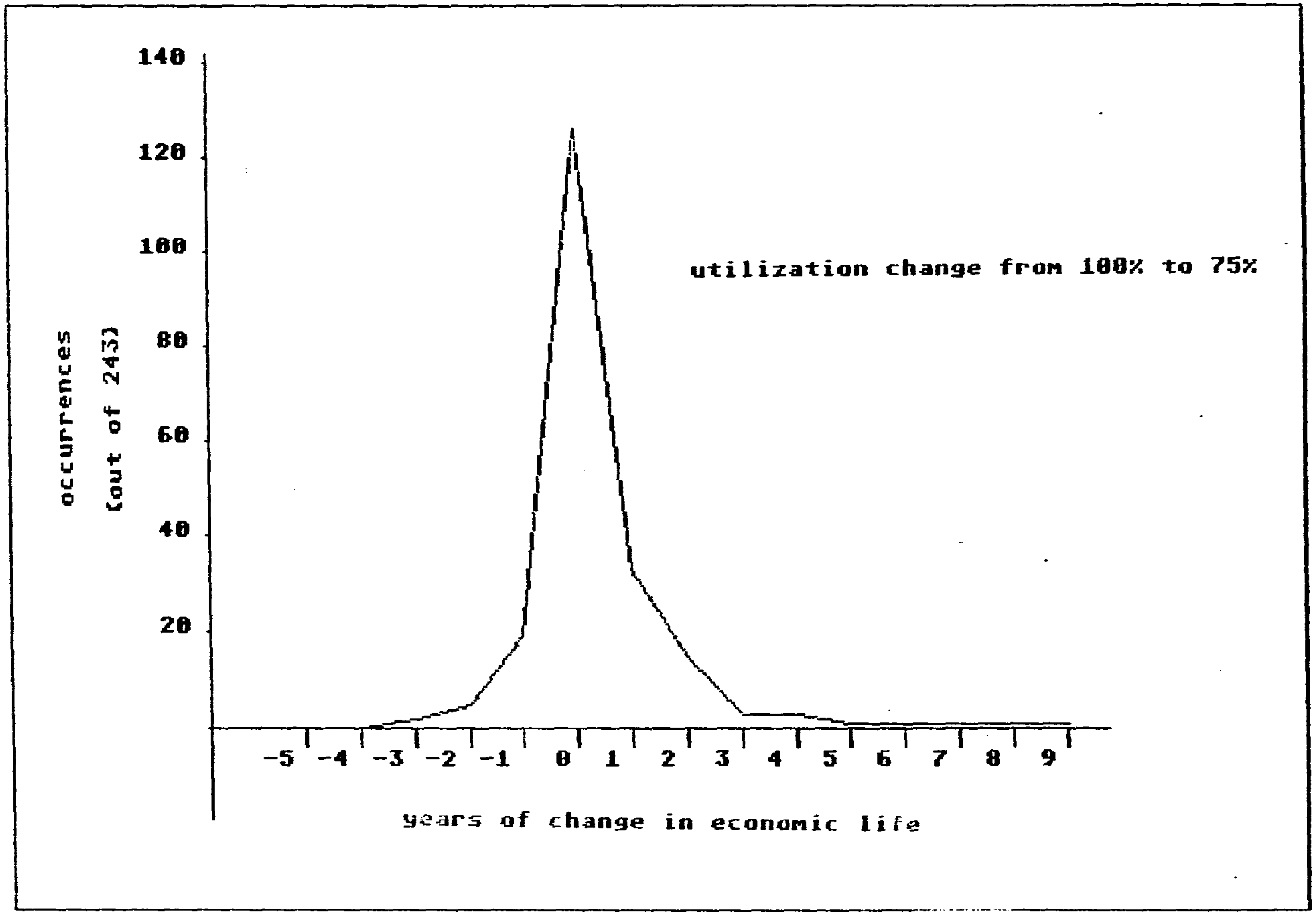

Figure 4.6: Occurrences of change in economic life when utilization drops from 100% to 75%

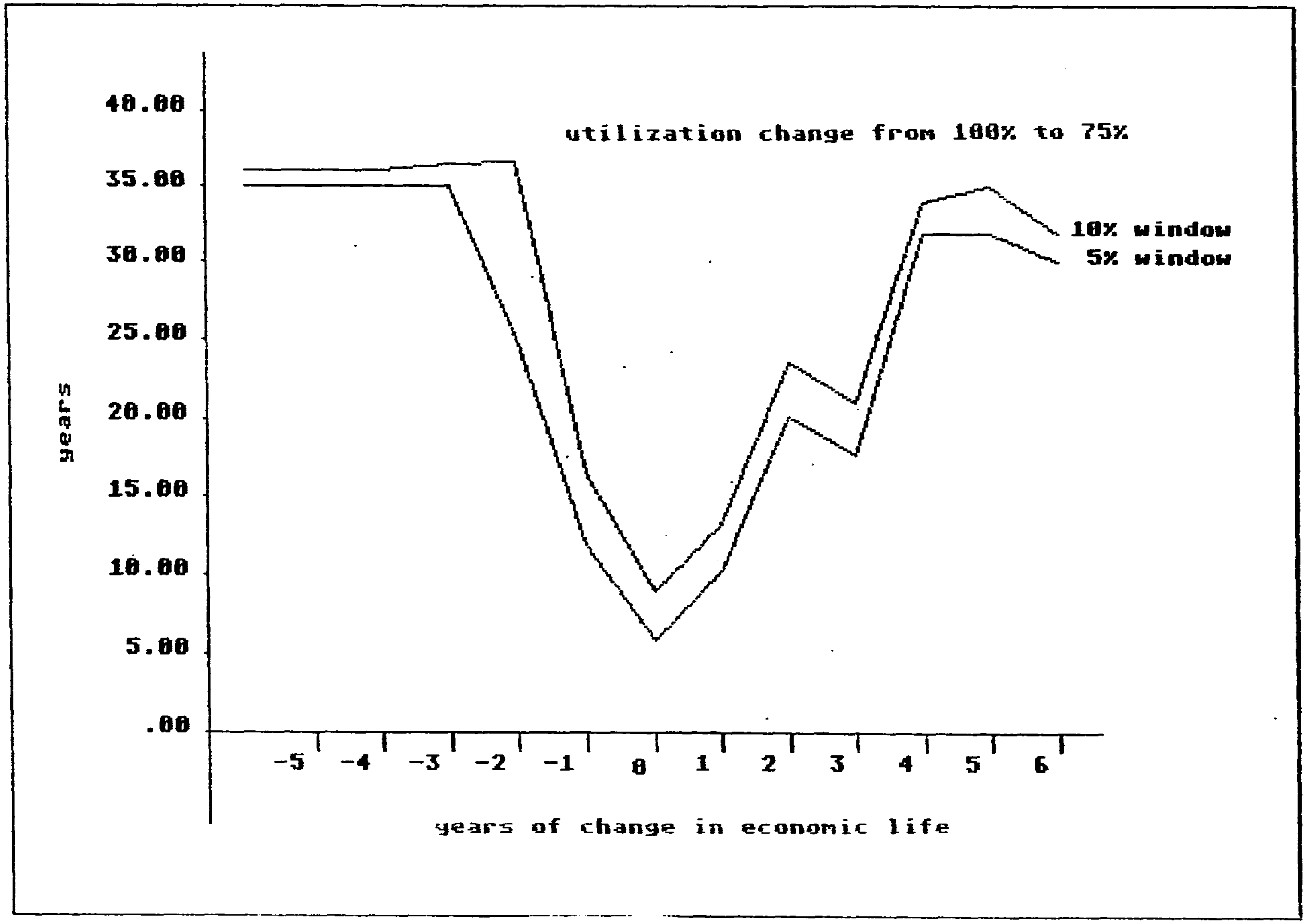

Figure 4.7: Width of 5% and 10% *AEC* window as a function of how much the economic life changes when utilization drops from 100% to 75%

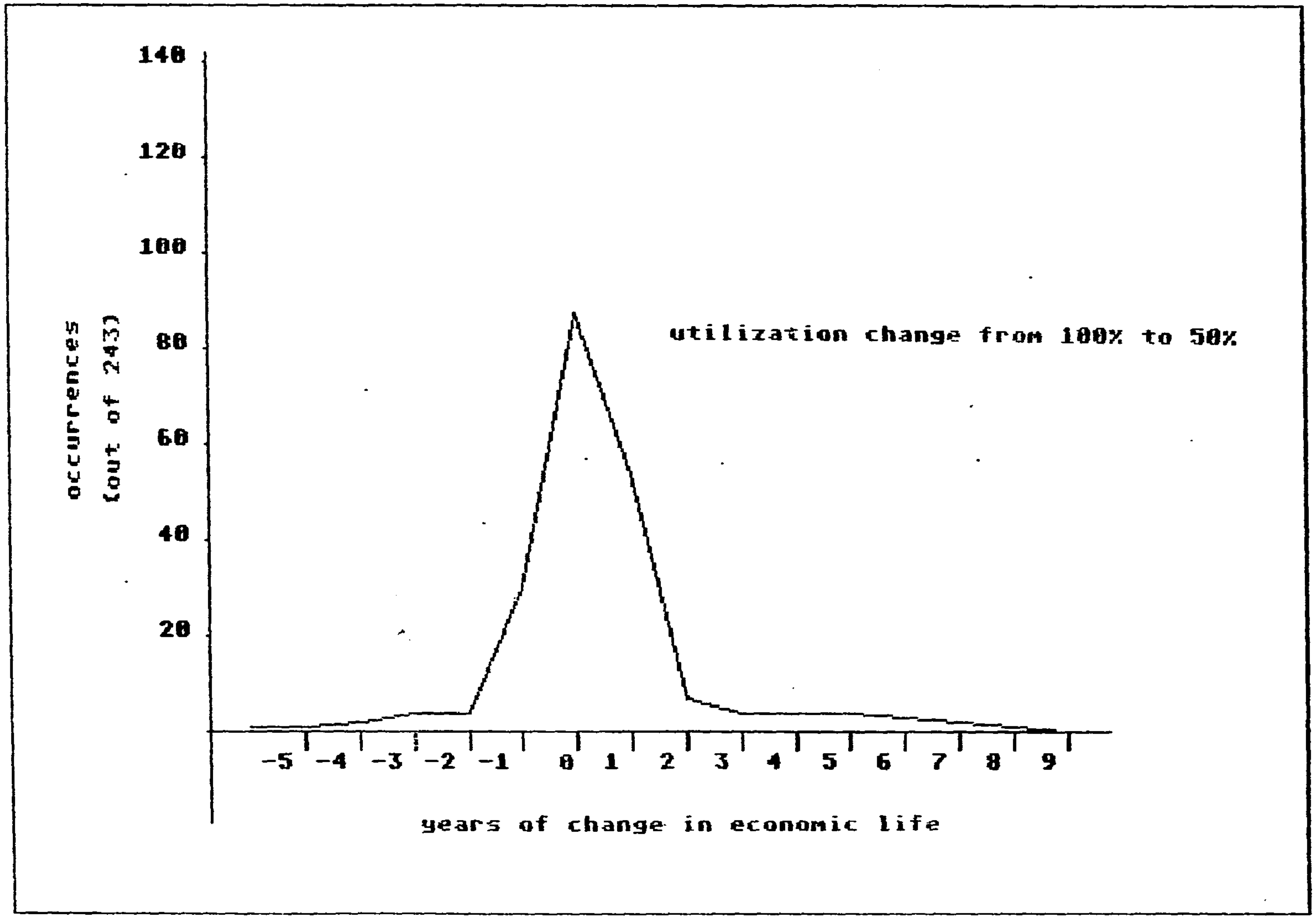

Figure 4.8: Occurrences of change in economic life when utilization drops from 100% to 50%

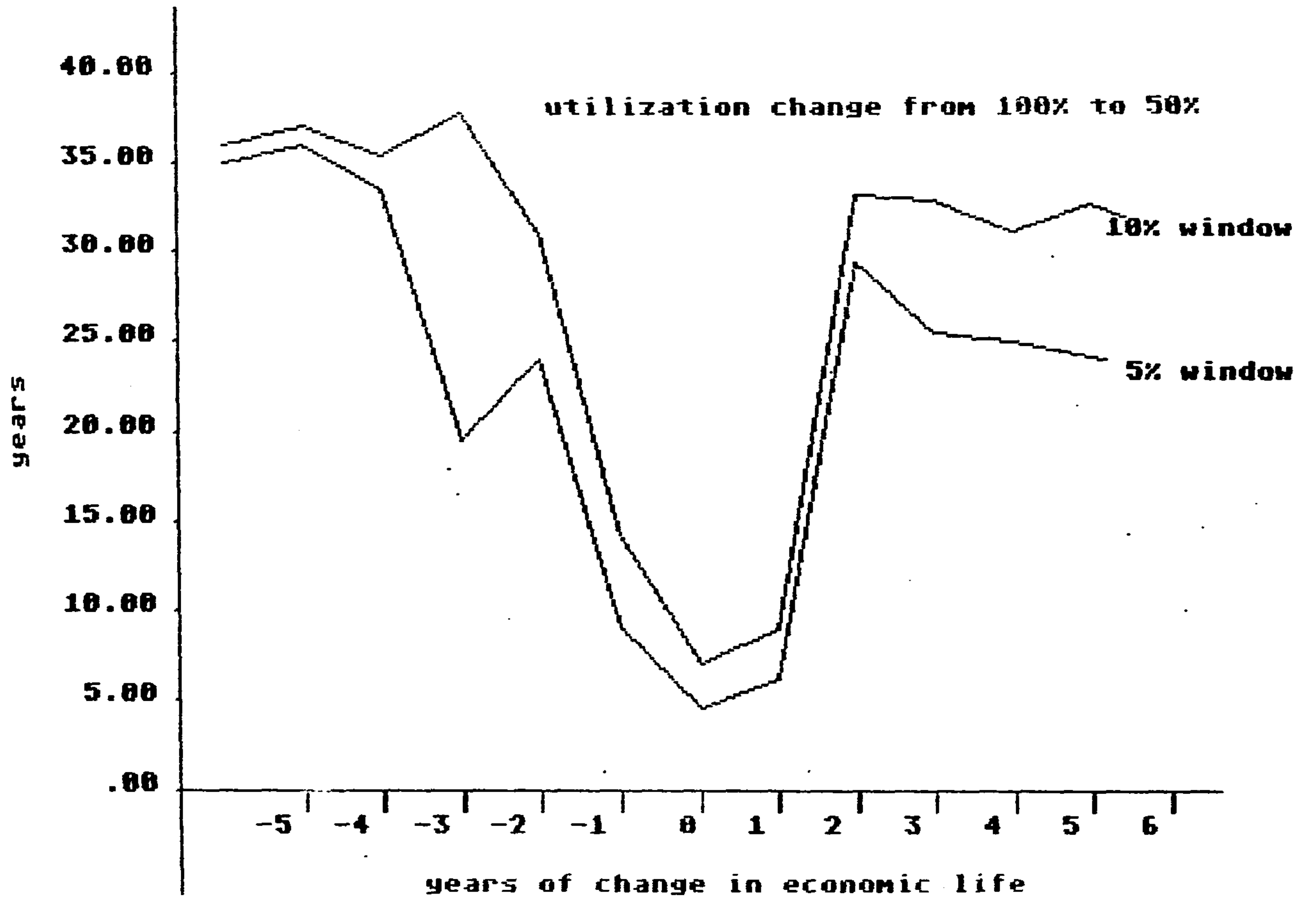

Figure 4.9: Width of 5% and 10% *AEC* window as a function of how much the economic life changes when utilization drops from 100% to 50%

time. With the demand for product produced being more than either machine can produce alone, but less than both can produce when both operating at 100%, how should the load be distributed between the two machines to minimize the machine costs?

Machine one has identical cost characteristics to the machine in the example above. The second machine has 50% more capacity, and has costs as described below:

$$B_2 = 50,000$$

$$V_2(n) = \frac{.5B_2}{(1+i)^{2n}} + \frac{.5B_2}{(1-i)^{2cu(n)}}$$

$$aroc_2(n) = 400(n-1)$$

$$UROC_2(n) = 1900cu(n)$$

From these data, the economic life, AEC, and normalized cost per unit are computed for machine 2. (See Table 4.4.) In this table, the normalized cost per unit is normalized with respect to machine one, taking into account the fact that machine two can produce 50% more than machine one at 100% utilization.

Let

C_k = the capacity of the kth machine

U_k = the utilization level of the kth machine

D = the demand of product

With two machines the following is true,

$$C_1U_1 + C_2U_2 = D \tag{4.4}$$

For the two machine example, let $C_1 = 1$, $C_2 = 1.5$. If the demand, D is 2, then various combinations of loading levels of the two machines can produce the required

Table 4.4: Economic life, AEC, and cost per unit for machine 2

U	economic life	AEC	normalized cost/unit
100%	8	18790	1.082
95%	8	18533	1.123
90%	8	18273	1.169
85%	9	18000	1.219
80%	9	17718	1.275
75%	9	17443	1.339
70%	10	17144	1.410
65%	10	16837	1.492
60%	10	16526	1.586
55%	11	16208	1.697
50%	11	15877	1.829

quantity. Table 4.5 above shows the AEC for each machine and for both together. Looking at the AEC values for this example, the cost is minimized by loading one machine at 100%, in this case, the best machine to fully utilize is machine 1.

With a rather logical assumption on the characteristics of minimum AEC as a function of U it can be shown that the optimal strategy is to load the machine with the lowest normalized cost per part to the 100% level.

Let the variables C_1, C_2, and D be defined as before. Let $f(u)$ be the minimum annual equivalent cost of machine 1 when operated at utilization level u. Let the function $g(u)$ be the minimum annual equivalent cost of machine 2 when machine 1 is operated at utilization level u. Assume that $f(u)$ is a monotonically increasing concave function in the region of utilization possible (given the capacities and demand). Assume that $g(u)$ is a monotonically decreasing concave function. (Note that $g(u)$ represents the costs of machine 2 as a function of machine 1 utilization. Since machine 2 utilization is inversely proportional to machine 1 utilization, the

Table 4.5: AEC as a function of machine utilization mix

U_1	U_2	AEC_1	AEC_2	TOTAL AEC
100%	67%	11577	16940	28517
95%	70%	11421	17144	28565
90%	73%	11257	17337	28594
85%	77%	11093	17528	28621
80%	80%	10926	17718	28644
75%	83%	10757	17904	28661
70%	87%	10576	18094	28670
65%	90%	10393	18273	28666
60%	93%	10208	18446	28654
55%	97%	10012	18619	28631
50%	100%	9814	18790	28604

costs of machine 2 are monotonically increasing as its own utilization goes up.)

To show that the minimum cost occurs where one of the machines is fully utilized, the initial assumption will be made that there does exist a utilization level, u', with total AEC less than that where either of the machines is operated at 100%. It will then be shown that this assumption leads to a contradiction.

Let L equal the lowest utilization possible on machine 1. This occurs when machine 2 is fully utilized. From equation 4.4 we find that:

$$L = \frac{D \times C_2}{C_1} \tag{4.5}$$

By assumption:

$$f(u') + g(u') < f(L) + g(L) \tag{4.6}$$

$$f(u') + g(u') < f(1) + g(1) \tag{4.7}$$

where

$$L < u' < 1 \tag{4.8}$$

Since the f function is monotonically increasing and is concave (the derivative is always increasing), then:

$$\frac{f(u') - f(L)}{u' - L} > \frac{f(1) - f(u')}{1 - u'} \tag{4.9}$$

Similarly, since g is monotonically decreasing and concave,

$$\frac{g(L) - g(u')}{u' - L} < \frac{g(u') - g(1)}{1 - u'} \tag{4.10}$$

Equation 4.6 can be rewritten as,

$$f(u') - f(L) < g(L) - g(u') \tag{4.11}$$

Equation 4.7 can be rewritten as,

$$f(1) - f(u') > g(u') - g(1) \tag{4.12}$$

From 4.9 and 4.11 it can be found that:

$$\frac{g(L) - g(u')}{u' - L} > \frac{f(1) - f(u')}{1 - u'} \tag{4.13}$$

From 4.10 and 4.12 it can be found that:

$$\frac{g(L) - g(u')}{u' - L} < \frac{f(1) - f(u')}{1 - u'} \tag{4.14}$$

Looking at equation 4.13 and equation 4.14 reveals a contradiction, the left hand quantity cannot be both less than and greater than the right hand quantity. Thus, the original assumption leads to a contradictory result and there can be no u' value that provides a lower total minimum AEC. (Note if we define the f and g functions to be strictly nondecreasing and strictly nonincreasing respectively, then equation

4.13 and equation 4.14 will not be strict inequalities and there can be a u' with a value the same as the endpoint, but not less than it.)

Before proceeding to the third example notice the sensitivity (or lack thereof) of total AEC to utilization mix in Table 4.5. With the values used in this example, the AEC value only ranges from 28,517 to 28,670, a change of only one half of a percent! This follows from the sensitivity study of AEC as a function of utilization done earlier for a single machine. If demand is fixed, and there are a number of machines which are able to perform a given operation, the exact loading of the machines may not be too critical from a pure cost standpoint. Other considerations such as scheduling availability, setup costs, and quality may be more significant factors than optimum AEC.

4.6 One Machine, Non-like-for-like Replacement, and/or Varying Service Need

If the like-for-like replacement assumption is removed or if the service requirement is varying, the approach of finding the n value that yields the minimum AEC is inadequate. Instead, any possible sequence of replacements within a fixed time interval must be evaluated. The work of this task can be greatly reduced using a dynamic programming technique.

Let the B, V, $aroc$, and $UROC$ functions defined above be subscripted to represent different machines available in different years. Thus,

$B(C,Y)$ = first cost of challenger C available in year Y.

$V(C,Y,N,cu(N))$ = the salvage value of of challenger C available in year Y if purchased and used for N years with $cu(N)$ cumulative years of use.

$aroc(C, Y, N)$ = the age related operating costs incurred in the Nth year of use of challenger C acquired in year Y.

$UROC(C, Y, N)$ = the use related operating costs incurred in the Nth year of use of challenger C acquired in year Y if the cumulative utilization at the end of year N is $cu(N)$.

Using these modified definitions the present equivalent costs to use a machine can be expressed similarly to equation 4.4.

$$PEC(C,Y,N,cu) = \Big(B(C,Y) - V(C,Y,N,cu(N))(P/F)^i_N + \sum_{n=1}^{N} (aroc(C,Y,n) + UROC(C,Y,n))\,(P/F)^i_n\Big) \tag{4.15}$$

Following the approach of Oakford, Lohmann, and Salazar [1984] a forward dynamic programming model can be formulated that will provide a prospectively optimal sequence of replacements. The recursive optimality equation is:

$$PEC^*(H) = \min PEC^*(j) + PEC(C, j, H - j + 1, cu)] \tag{4.16}$$

$$j = 0, 1, 2, ...H - 1, \quad C = 1, 2, ...number\ of\ alternatives$$

The optimal machines to be used can be signified by,

$Y^*(H)$ = the year of purchase of the last machine in the sequence of prospectively optimal replacements,

and

Table 4.6: First costs, capacity, and demand

n	$B(1,n)$	$C(1,n)$	$B(2,n)$	$C(2,n)$	$D(n)$
0	10,000	125	30,000	150	60
1	30,000	125	35,000	175	70
2	30,000	125	40,000	200	75
3	29,000	150	37,000	200	80
4	29,000	150	35,000	200	90
5	23,000	150	35,000	200	100

$C^*(H)$ = the challenger number of the last challenger in the sequence of prospectively optimal replacements.

A sample problem is now shown to demonstrate this methodology. The first costs, capacities, and demand are shown in Table 4.6. For the defender (challenger 1 in year 1) the salvage value is:

$$V(1,0,n,cu) = .5B(1,0)\left(1-\frac{cu}{10}\right) + .5B(1,0)\left(1-\frac{n}{10}\right)$$

For the other challengers, the salvage value is:

$$V(C,m,n,cu(n)) = \left(\frac{.4B(C,m)}{(1+i)^{2n}}\right) + \left(\frac{.4B(C,m)}{(1+i)^{2cu}}\right)$$

The year by year maintenance costs for the defender are:

$$aroc(1,0,n) = 8000 + 1300n$$

$$UROC(1,0,n) = 2800 * (cu(n)^2 - cu(n-1)^2)$$

The year by year maintenance costs of the challengers are:

$$\begin{aligned} aroc(C,m,n) &= 2000 + 1300n \quad & m &= 2,3 \quad & C &= 1 \\ &= 2100 + 1200n \quad & m &= 1,2,3 \quad & C &= 2 \\ &= 2000 + 1000n \quad & m &= 4,5,6 \quad & C &= 1 \\ &= 800 + 700n \quad & m &= 4,5,6 \quad & C &= 2 \end{aligned}$$

$$
\begin{aligned}
UROC(C,m,n) &= 2000\,(cu(n)^2 - cu(n-1)^2) & m &= 2,3 \\
& & C &= 1 \\
&= 1700\,(cu(n)^2 - cu(n-1)^2) & m &= 1,2,3 \\
& & C &= 2 \\
&= 1400\,(cu(n)^2 - cu(n-1)^2) & m &= 4,5,6 \\
& & C &= 1 \\
&= 1100\,(cu(n)^2 - cu(n-1)^2) & m &= 4,5,6 \\
& & C &= 2
\end{aligned}
$$

The $cu(n)$ function is calculated from the machine capacities and the demand. If $C(j)$ represents the machine used in year j in some pattern of replacements, then the cumulative hours used can be expressed by,

$$cu(n) = \sum_{j=1}^{n} cap\left(\frac{C(j)}{D(j)}\right)$$

The interest rate again is 20% and a before tax approach is used. Table 4.7 shows the PEC values that were calculated for all lengths of equipment life for all challengers. The defender is denoted by $C = 1$, starting at year 1. Table 4.8 shows the optimal sequence of machines using the terminology described earlier.

From Table 4.8 it can be seen that if the horizon is 3 years, purchase the challenger machine at the beginning of year 1. If the horizon is 6 years, then start with machine 1 and replace it with challenger 1 in year 4.

For comparison purposes, consider the changes in the results if there were forecast a slower rise in demand. Let the demand for each year in the six year horizon be as in Table 4.9. The optimal sequence changes to that shown in Table 4.10.

Table 4.7: $PEC(C, START, USED)$

	$C = 1$					
Start	Used years					
year	1	2	3	4	5	6
1	10571	21415	32352	43241	54243	65339
2	15146	23686	31316	38604	45779	
3	12757	20041	26762	33274		
4	9931	15259	19866			
5	8410	12995				
6	5900					

	$C = 2$					
Start	End year					
year	1	2	3	4	5	6
1	17389	26705	34652	41712	48294	54559
2	16493	24873	31825	37941	43506	
3	15320	22835	29012	34326		
4	11095	16122	20040			
5	8889	12989				
6	7497					

Table 4.8: Optimal sequence of equipment replacements

H	$PEC^*(H)$	$Y^*(H)$	$C^*(H)$
1	10570.93	1	1
2	21414.93	1	1
3	32351.51	1	1
4	41455.98	3	1
5	47610.98	4	1
6	52218.00	4	1

Table 4.9: Alternate demand forecast

Year	Demand
1	80
2	100
3	110
4	120
5	135
6	150

Table 4.10: Optimal sequence for faster rise in demand

H	$PEC^*(H)$	$Y^*(H)$	$C^*(H)$
1	11055.73	1	1
2	23552.40	1	1
3	37302.77	1	1
4	45427.10	2	2
5	52806.22	2	2
6	58927.23	4	2

4.7 More Than One Machine, Non-like-for-like Replacement, and/or Varying Service Need

Now the analysis is extended to the situation where the capacity of one machine is insufficient to meet the required demand. Two machines are required to generate the needed product. We will assume that no overtime is allowed, that the value of a machine at the end of the fixed horizon time is the salvage value. C different machines are available each year for purchase to replace one or both of the currently used machines.

The assumption is made that one of the machines will be run at 100% and the other machine will produce to meet the remaining demand. Call the machine that is producing at 100% machine A, the other, machine B. The PEC of machine A

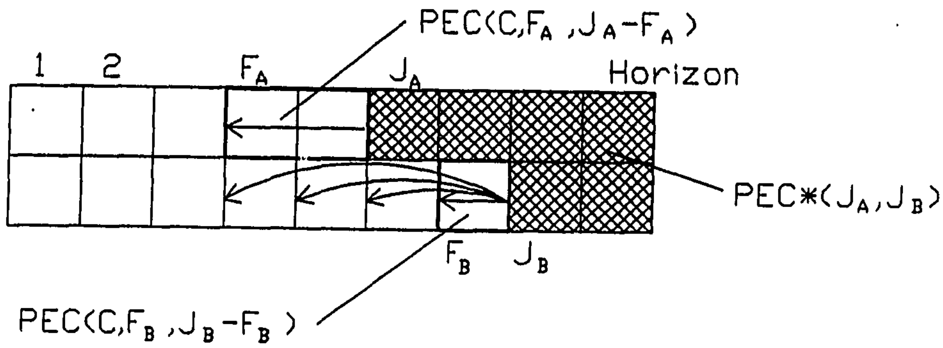

Figure 4.10: The two dimensional dynamic programming stages

can be computed independently of B. It will be the *PEC* at 100% utilization for whatever lifetime it is used. The *PEC* of a B machine will depend on which A machine or machines it operates in conjunction with over its lifetime.

The problem can be solved using a two-dimensional dynamic programing algorithm. Each dimension corresponds to one of the two machines. At each stage in the "outside" dimension the optimal sequence of replacements is found to supply A machine service for a given period of time ending at the planning horizon and B machine service for any period of time shorter than or equal to the machine A time period and also ending at the planning horizon. The "inner" dimension steps through each of the shorter B machine life periods. See Figure 4.10.

A backwards directed approach is used rather than the previously used forward

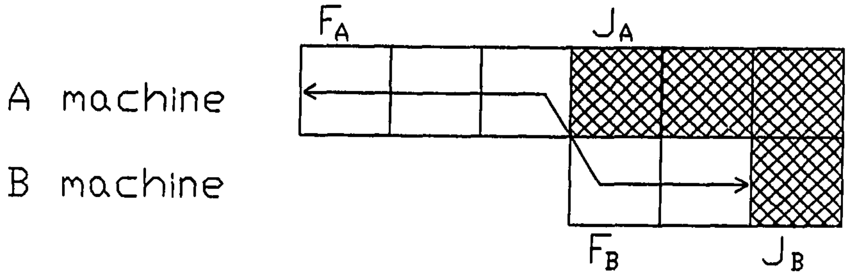

Figure 4.11: A replacement sequence of a demoted machine

approach for the following reason. The cost of a B machine can only be computed if the capacity of the A machine used with it, during each year of its life, is known. This fact requires that the stages must proceed with the period of B machine service always less than or equal to the A machine period. When a new item of equipment is purchased, one of the two machines being used is disposed of and the other machine, if currently serving as the A machine, can be demoted to secondary or B machine use. This creates a pattern as shown in Figure 4.11. Such a pattern cannot be included if a forward approach is used, since this requires both the A and the B machine period to start at the same time, but can be included in the backwards approach which requires that both periods end at the same time.

The recursive optimality formulation that follows allows for this demotion from primary to secondary service when a machine is replaced. Three cumulative utilization functions are defined to model this.

cu_A = the cumulative use function for a machine that is operated at 100% utilization.

cu_B = the cumulative use function of a machine that supplies the remaining demand in years 1 though n.

$cu_{AB}(S)$ = the cumulative hours used for a machine that is operated at 100% utilization during the first S years of its life and later demoted to less than 100% utilization.

The function $PEC(C, Y, N, cu)$ is defined as in equation 4.16. Let $PEC^*(F_A, F_B)$ be the optimal PEC of a pattern of replacements that provides A machine service for years F_A through the horizon time, H, and B machine service for years F_B through H. PEC^* again, is only defined for cases where $F_B \geq F_A$. The optimal machines and their purchase years are described by the following functions:

$Y_A^*(H)$ = the year of retirement of the first A machine in the sequence of prospectively optimal replacements starting in year H

$Y_B^*(H)$ = the year of retirement of the first B machine in the sequence of prospectively optimal replacements starting in year H

$C_A^*(H)$ = the challenger number of the first A machine in the sequence of prospectively optimal replacements starting in year H

$C_B^*(H)$ = the challenger number of the first B machine in the sequence of prospectively optimal replacements starting in year H

The recursive optimality equation is:

$$PEC^*(F_A, F_B) = \min(PEC^*(J_A, J_B) + PEC_{AB}(J_A, J_B, F_A, F_B)) \qquad (4.17)$$

where

$$PEC_{AB}(J_A, J_B, F_A, F_B)$$

$$= PEC(C, F_A, J_A - F_A, cu_A) + PEC(C, F_B, J_B - F_B, cu_B) \qquad (4.18)$$

if $C = 1, 2, ..number\ of\ alternatives; J_A = H, H-1, ...F_A; J_B = H, H-1, ...J_A$ and

$$PEC_{AB}(J_A, J_B, F_A, F_B) = PEC(C, F_A, J_B - F_A, cu_{AB}(J_A - F_A)) \qquad (4.19)$$

if $C = 1, 2, ..number\ of\ alternatives; J_A = F_B; J_B = F_B + 1, ...H$

The following algorithm is used.

```
For $F_A$ = HORIZON to 1 by -1
  For $F_B$ = HORIZON+1 to $F_A$ by -1
    For $J_A$ = HORIZON+1 to $F_A$ by -1
      For $J_B$ = HORIZON+1 to max($J_A$,$F_B$)
        For $C_1$ = 1 to machines available in year $J_A$
          For $C_2$ = 1 to machines available in year $J_B$
            compute $cu_B$ from demand and A machine capacity
            $P = PEC^*(J_A, J_B) + PEC(C_1, F_A, F_A - J_A, cu_A)$
                  $+PEC(C_2, F_B, F_B - J_B, cu_B)$
            If $P < PEC^*(F_A, F_B)$ then
              save $P$ as new $PEC * (F_A, F_B)$ value
              save the machine numbers $C_1$ and $C_2$
              save the year number of the replacements
            endif
```

```
        If $J_A = F_B$ and $J_B = F_B$ then
          compute $cu_{AB}$
          $P = PEC^*(J_A, J_B)$
              $+PEC(C_1, F_A, J_B - F_A, cu_{AB}(J_A - F_A))$
          If $P < PEC(F_A, F_B)$ then
            save P as new $PEC*(F_A, F_B)$ value
            save the machine numbers $C_1$ and $C_2$
            save the year number of the replacements
          endif
        endif
end all loops
```

Figure 4.12 below shows pictorially the replacement patterns that would be computed to find the minimum cost path given that the planning horizon is 6 years, F_A is 2, and F_B is 4. The eleven patterns on the left are those patterns where there is no demoting of the machines that are not already in a previously found optimal pattern. The three patterns on the right are the possible demotion patterns for this example.

Given the same data as in the last example, except that the demand is changed to that given in Table 4.11, the algorithm generates the results shown in Table 4.12. In this table are shown the optimal cost and the year and challenger number of the most recent replacement for any given start year of machine A (across the top) and start year of machine B (down the table). Also, above the optimal cost is a graphical representation of the service years provided with A machine service above

B machine service. Thus the optimal cost to provide service for the entire time horizon is $119,815. This cost occurs if the following sequence of replacements are made.

Purchase two challenger 1s in the first year. In year three retire the one that has been used to provide *B* machine service and demote the one that has provided *A* service to *B* service. Purchase challenger 1. In year four, again purchase challenger 1 and retire the *B* machine, demote the *A* machine. In year 6, do the same one more time.

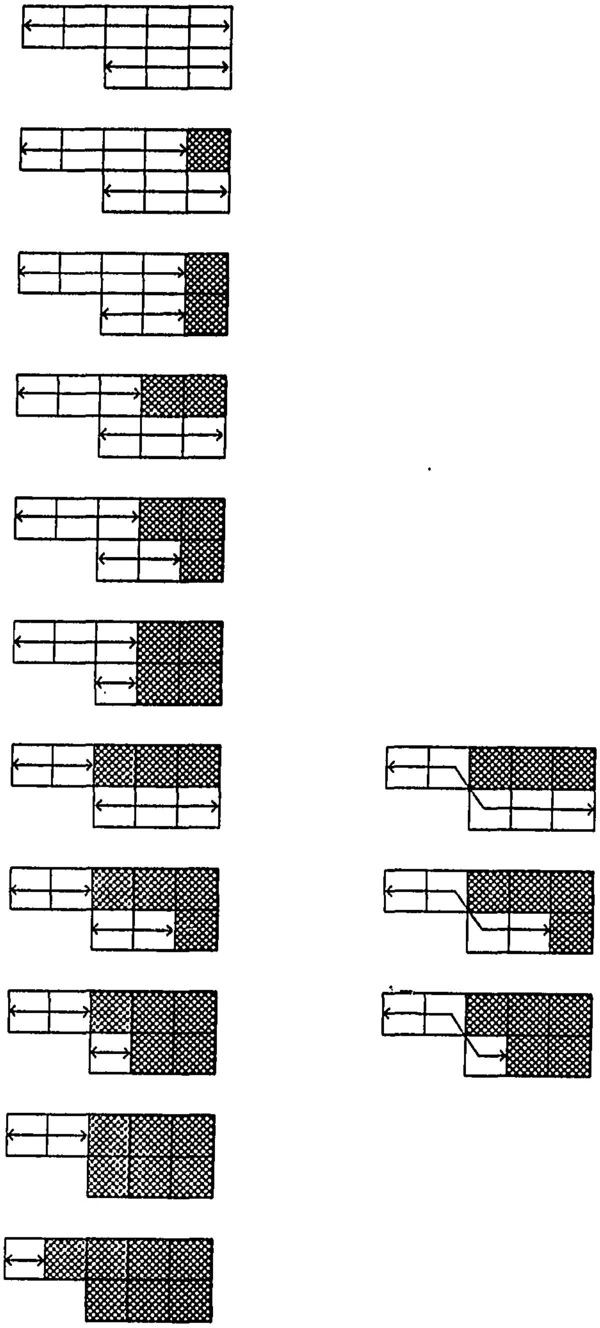

Figure 4.12: Replacement patterns for a 6 year planning horizon when $F_A = 2$ and $F_B = 4$

Table 4.11: Revised demand forecast

Year	Demand
1	200
2	230
3	260
4	300
5	300
6	300

Table 4.12: Results of the two dimensional dynamic programming example

	7	6	5	4	3	2	1
7	0 0/ 0 0/ 0	* 6437 7/ 1 7/ 0	** 14753 7/ 1 7/ 0	*** 23616 7/ 1 7/ 0	**** 37872 4/ 1 7/ 0	***** 51733 4/ 1 7/ 0	****** 64317 2/ 1 7/ 0
6		* * 12466 6/ 1 -7/ 1	** * 20572 6/ 1 -7/ 1	*** * 29256 6/ 1 -7/ 1	**** * 43511 4/ 1 6/ 0	***** * 57373 4/ 1 6/ 0	****** * 69956 2/ 1 6/ 0
5			** ** 28269 5/ 1 -7/ 1	*** ** 36732 5/ 1 -7/ 1	**** ** 50987 4/ 1 5/ 0	***** ** 64849 4/ 1 5/ 0	****** ** 77432 2/ 1 5/ 0
4				*** *** 44757 7/ 2 7/ 2	**** *** 58744 4/ 1 -6/ 1	***** *** 72874 4/ 1 4/ 0	****** *** 85457 2/ 1 4/ 0
3					**** **** 71263 3/ 1 -6/ 1	***** **** 85057 3/ 1 -4/ 1	****** **** 97640 2/ 1 3/ 0
2						***** ***** 97527 2/ 1 -4/ 1	****** ***** 109840 2/ 1 -3/ 1
1							****** ****** 119815 1/ 1 -3/ 1

5 INTEGRATION MODELING

The model presented in this chapter is a simplified model of costs, revenues, and product quality in a manufacturing system. The objective is to show qualitatively what such an approach indicates about replacement decisions and replacement decision analysis. Three aspects of the model are of particular importance.

1. as in Chapter 4 the degradation of equipment is a function of both age and use.
2. the characteristics of each component of the system have an effect on the operation of the other components.
3. the product demand is a function of the manufacturing system's characteristics.

There are many interactions of equipment items in a system. For simplicity, the sole interaction modeled here is that caused by defects created during the manufacturing process. This affects the work load of other equipment and thus the costs. It also has an effect on the product's reputation in the market place, which impacts the product demand.

5.1 The Illustrative Model

The replacement decision criterion is present equivalent worth of net cash flows within a fixed time horizon. End of year cash flows and discrete compounding are used. The model is developed from a before-tax perspective.

A set of formulas describes the computation of the present equivalent of the system aggregate worth ($PESAW$) for a fixed time horizon. Replacement decisions that lead to a positive value of $PESAW$ are considered acceptable. Those decision sets yielding $PESAW < 0$ are unacceptable. The system is a collection of M processors, where "processor" is a generic term denoting an entity which performs an operation in the manufacturing process. To make the model as broad as possible, an operation is defined to be any action taken in the manufacturing of the product, whether a value-added operation or not. Thus "operation" includes the order given by the manager to commence production of a lot of parts, the material handling necessary to move parts from one machine to another, as well as drilling, casting, or assembly tasks. Correspondingly, a "processor" can be a machine, a conveyor belt, a manager, or a section of factory floor where work in process is stored. The system produces various products which are made up of component parts described by a parts explosion and process routings.

5.1.1 Notation

$PESAW$ = present equivalent of system aggregate worth

H = the planning horizon in years

P = number of different products the company produces

$R(p)$ = the number of component parts in product p

M = the number of processors in the system

$PAR(n,p)$ = product annual revenues in year n for product p

$ARM(n,p,r)$ = annual raw material cost in year n for part r of product p

$PEMRC(m)$ = present equivalent of processor related costs for processor m

$Q(n,p,r,o)$ = starting quantity in year n for operation o of part r of product p

$UP(n,p)$ = unit selling price in year n or product p

$UMC(n,p,r)$ = unit raw material cost in year n of part r of product p

$V(m,n,u)$ = resale or salvage value of processor m with age n and cumulative use of u hours

$I(m)$ = installation cost of processor m beyond price of equipment

$DV(m,n,u)$ = disposal value of processor m with age n and cumulative use of u hours. $PV(m,n,u)$ if horizon ends before machine is retired, else $= V(m,n,u)$

$PV(m,n,u)$ = the potential value of processor m with age m and cumulative use of u hours. This is the value in terms of what benefits the equipment can provide minus its costs.

l = the life of the processor in years

$cu(m,n)$ = the cumulative time used on processor m at the end of year n

$aroc(m,n)$ = age related operation costs of processor m in year n

$uroc(m, cu)$ = use related operation cost rate of processor m after cu cumulative hours of use

$UROC(m, n)$ = cumulative use related operating cost of processor m in year n

$O(p, r)$ = number of operations needed to make part r of product p

$OT(p, r, o)$ = operation time for operation o on part r of product p

$comp(r)$ = the part into which part r is assembled

$MF(m, n, p, r, o)$ = 1 if operation o of part r of product p is performed by processor m in year n, 0 otherwise

$QRM(p, r)$ = quality of raw material (percentage of bad material) used for part r of product p

$GB(m, n)$ = fraction of parts that start good and become bad on processor m in year n

$BG(m, n)$ = fraction of parts that start bad and become good on processor m in year n

$BD(m, n)$ = fraction of parts that are discarded on processor m in year n

$g(n, p, r, o)$ = the percentage of good part r of product p remaining after operation o in year n

$b(n, p, r, o)$ = the percentage of bad part r of product p remaining after operation o in year n

$d(n,p,r,o)$ = percentage of parts that have been discarded up through operation o on part r of product p in year n

$BP(n,p)$ = the percentage of product p that leaves the factory in a flawed condition in year n

$D(p,n)$ = the demand for product p in year n

$MS(p,n)$ = the firm's market share for product p in year n

$PP(p,r)$ = the quantity of part r in one unit of product p

$Fms(BP)$ = the function that relates the product quality history, BP, to the firm's market share

5.1.2 Computation of $PESAW$

The present equivalent system aggregate worth is equal to the discounted sum of the revenues minus material costs minus the processor related costs. This is expressed in equation 5.1.

$$PESAW = \sum_{N=1}^{H} \left(\sum_{p=1}^{P} \left(PAR(n,p) - \sum_{r=1}^{R(p)} (ARM(n,p,r) \right) \right) \cdot (P/F)_n^i - \sum_{m=1}^{M} PEMRC(m) \tag{5.1}$$

5.1.3 Revenues and raw material costs

Revenues and raw materials are based on quantity sold and quantity started in production, respectively. These two quantities differ by the amount of material that is scrapped due to defects. For simplicity, it is assumed that the quantity produced equals the quantity sold equals the quantity demanded. The final assembly of any product is numbered as part number 1. The product annual revenue in year n is given by,

$$PAR(n,p) = Q(n,p,1,O(p,1)+1) \times UP(n,p) \tag{5.2}$$

Using $O(p,1)+1$ as the operation number means the starting quantity of the operation after the last one. In other words, this is the ending quantity of the last operation.

The annual raw material cost is that of the quantity of parts started in production. Thus,

$$ARM(n,p,r) = Q(n,p,r,1) \times UMC(n,p,r) \tag{5.3}$$

5.1.4 Processor related costs

The value of a processor is modeled as a function of age and cumulative use. V represents the resale value of the equipment in contrast to PV which is the value in terms of what benefits the processor can provide to the firm. This is the distillate of the expected revenue generated minus the costs incurred in the use of the machine during its remaining life. The symbol DV represents the disposal value, and is either V if the machine is sold before the end of the horizon or $PV(m,a,u)$ if the machine is to be kept longer than the planning horizon.

The operating costs are also modeled as a function of age and usage. As in Chapter 4, $aroc(m,n)$ represents the age related cost in year n to operate processor m. The use related operating costs, $uroc(m,u)$, is the cost per unit time of use as dependent on the cumulative time for which the machine has provided service to date. The unit time used in Chapter 4 was one year. In this chapter the time unit used is one hour. This is more appropriate for cumulating processing time on various machines. Although *uroc* is modeled as a continuous function, the cash flows involved are treated as end of year amounts, since the timing of the machine usage during each year is not determined by the model. Thus the cumulative function, $UROC$, is as described in Chapter 4.

Assume that the processor is acquired in year f and retained for l years. The present equivalent of the processor related costs is given by equation 5.4.

$$PEMRC(m) = \Big(V(m,0,0) + I(m) - DV(m,l,cu(m,l))(P/F)_l^i + \sum_{n=1}^{l}\left(aroc(m,n) + \int_{cu(m,n-1)}^{cu(m,n)} uroc(m,n)du\right)(P/F)_n^i\Big)(P/F)_f^i \qquad (5.4)$$

The cumulative utilization in hours cu, is computed by totaling the hours required of each processor to perform the operations to produce the demanded quantity of end products.

$$cu(m,n) = \sum_{j=1}^{n}\sum_{p=1}^{P}\sum_{r=1}^{R(p)}\sum_{o=1}^{O(p,r)} Q(j,p,r,o) \times OT(p,r,o) \times MF(m,j,p,r,o) \qquad (5.5)$$

This is the total time needed for any processor to perform all the operations for all parts it processes in year 1 through year n. The cu(m,n) values are best calculated as a group for each year. The following algorithm is used.

```
Set all elements of $cu$ to 0
For $j = 1$ to $n$
 $cu(m,n) = cu(m,n-1)$
 For $p = 1$ to $P$
  For $r = 1$ to $R(p)$
   For $o = 1$ to $O(p,r)$
    $m$ = processor that performs operation $o$
    $cu(m,n) = cu(m,n) + Q(j,p,r,o) \times OT(p,r,o)$
   end all loops
```

5.1.5 Quality characteristics

The quality characteristics are modeled using expected values. $GB(m,n)$ is the probability of an operation on processor m producing a defect in a good part in year n. $BG(m,n)$ is the probability of noting a defect in a part and correcting it. $BD(m,n)$ is the probability of detecting a bad part and scrapping it. The average percentage of good, bad, and discarded component parts are given by equations 5.6, 5.7, and 5.8 respectively.

$$g(n,p,r,o) = g(n,p,r,o-1) \times (1 - GB(m,n)) + b(n,p,r,o-1) \times BG(m,n) \tag{5.6}$$

$$b(n,p,r,o) = b(n,p,r,o-1) \times (1 - BG(m,n) - BD(m,n)) + g(n,p,r,o-1) \times GB(m,n) \tag{5.7}$$

$$d(n,p,r,o) = \sum_{k=1}^{o-1} b(n,p,r,k) \times BD(m,n) \tag{5.8}$$

For the special case when o=0, the quality of product depends on the quality of the raw material or the quality of the component parts. These relationships are shown in equations 5.9 through 5.12.

$$b(n,p,r,0) = QRM(p,r) \qquad r = purchased\ part \tag{5.9}$$

$$g(n,p,r,0) = 1 - QRM(p,r) \qquad r = purchased\ part \tag{5.10}$$

$$g(n,p,r,0) = \prod_{k=all\ component\ parts\ of\ r} g(n,p,k,O(p,k)+1) \quad r = make\ part \tag{5.11}$$

$$b(n,p,r,0) = 1 - g(n,p,r,0) \qquad r = make\ part \tag{5.12}$$

The final percentage of bad product, $BP(n,p)$ is

$$BP(n,p) = \frac{b(n,p,1,O(p,1)+1)}{g(n,p,1,O(p,1)+1)+b(n,p,1,O(p,1)+1)}. \tag{5.13}$$

5.1.6 Quantity relationships

To find quantities it will be assumed that the production level just meets the demand. To calculate the quantities for each operation in the production process, first the ending quantities are found and then the quantities at each operation throughout the production process are computed based on the ending quantity and the quality characteristics of the processors. The ending quantity is either the demand, when the part is the final assembly, or the starting quantity for the higher level subassembly part in the parts explosion multiplied times the quantity of the lower level part required in the subassembly. This is shown in equations 5.14 and 5.15 below.

$$Q(n,p,r,O(p,r)+1) = D(p,n) \times MS(p,n) \qquad for\ r = 1 \tag{5.14}$$

$$Q(n,p,r,O(p,r)+1) = Q(n,p,comp(r),1) \times PP(p,r) \quad for\ r \neq 1 \tag{5.15}$$

The starting quantity of any given part, $Q(n,p,r,1)$, can be calculated when the ending quantity is known using equation 5.16.

$$Q(n,p,r,1) = \frac{Q(n,p,r,O(p,r)+1)}{1-d(n,p,r,O(p,r)+1)} \tag{5.16}$$

Finally, quantities at the beginning of any operation o can be computed from the relationship

$$Q(n,p,r,o) = Q(n,p,r,1) \times (1 - d(n,p,r,o)). \tag{5.17}$$

5.1.7 Market share function

Let the market share of a product be a function of the quality of product produced in previous years as in equation 5.18.

$$MS(n,p) = Fms\,(BP(n-1,p), BP(n-2,p), BP(n-3,p), \ldots) \tag{5.18}$$

5.2 Comparing Two Alternatives

Given a scenario of processors, products to produce, and market demand, $PESAW$ can be calculated. This example will compare two scenarios. First $PESAW$ will be computed given that the currently owned processors will be retained for the entire 5 year planning horizon. Compared to this will be the $PESAW$ if one of the processors is replaced at the beginning of year 1.

5.2.1 Processor data

A processor and its possible replacement are described in Table 5.1. The data of the other processors in the system is given in Appendix B. The currently owned

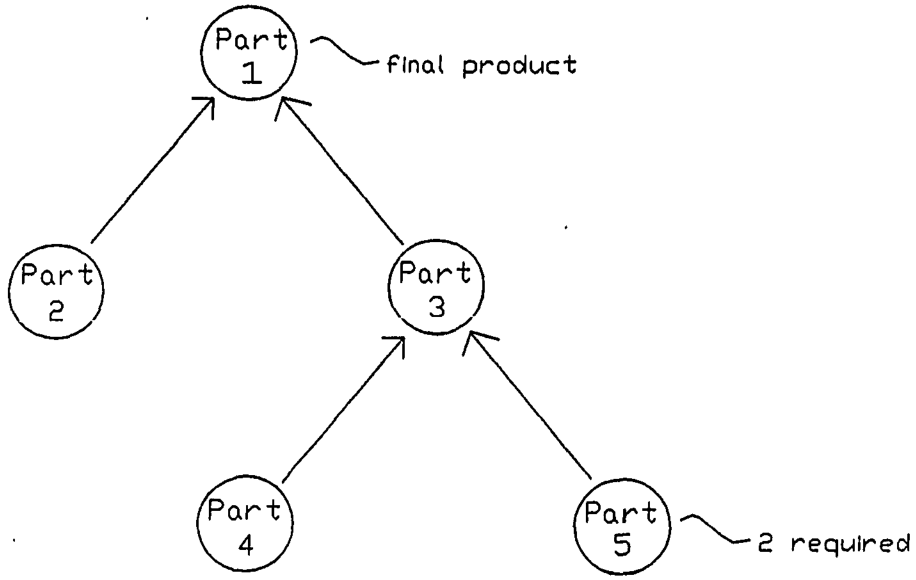

Figure 5.1: Parts explosion diagram of firm's sole product

machine has no installation cost since it is already in place. Note that the age, n, is in years, while u, the cumulative use, is in hours.

The operations are numbered with a five digit code that is used in the process routings. The operation times are given in hours per operation. Operations are performed either once per part or once per lot.

5.2.2 Product data

Figure 5.1 shows the parts explosion diagram for the firm's sole product. Part 1 is the finished product. There is one of each part in the upper level assemblies, except for part 5, of which two are needed to make part 3.

Table 5.1: The defender and challenger characteristics

	Defender	*Challenger*
I	0	8000
V	$\frac{22000}{n+6} + \frac{48000}{.0005u+6}$	$\frac{22000}{n+1} + \frac{48000}{.0005u+1}$
PV	$6800 + V$	$6800 + V$
$aroc$	$27000 + 3000n$	$22000 + 2500n$
$UROC$	$8(u + 10000) + \left(\frac{u+10000}{16}\right)^{1.45}$	$8u + \left(\frac{u}{12}\right)^{1.45}$
GB	$.045 + .000003.5u$	$.04 + .0000003.5u$
BG	.15	.15
BD	.25	.25
Operation	*times*	
02 – 001	25.00	23.00
02 – 003	.0027	.0021
02 – 004	.0030	.0025
02 – 005	.0028	.0019
02 – 021	25.00	23.00

Table 5.2: Demand for final product in entire market

Year	Expected Demand	Expected Lots
1	300,000	12
2	300,000	12
3	315,000	12
4	321,000	13
5	330,000	13

5.2.3 Market demand

The expected market demand is shown in Table 5.2. The firm currently maintains about a one-third market share. Based on this, the expected number of lots to produce each year is included in the table.

5.2.4 Process routings

In Table 5.3 are given the process routings for the five component parts of the firm's one product. The operation numbers are used to determine which processor can provide the function. The quantities are multiplied by the operation times in the processor data. Since some operations, such as setup, are done only once per lot, the routings include a column stating whether the operation occurs once per part or once per lot.

5.2.5 Raw material cost and quality

The data describing the incoming raw material and the expected quality are given in Table 5.4.

Table 5.3: Operations and quantities in the production of the product

Part	Op #	Operation	Quantity	Per lot/part
1	1	01-001	1	lot
	2	04-001	.0033	part
	3	02-005	.625	part
	4	03-001	1	lot
	5	01-001	1	lot
	6	02-021	1	lot
	7	10-001	5	part
2	1	02-001	1	lot
	2	01-001	1	lot
	3	02-004	.733	part
3	1	01-001	1	lot
	2	09-015	4	part
	3	03-031	15	lot
	4	03-001	3	lot
	5	02-004	1.11	part
4	1	02-001	1	lot
	2	01-001	1	lot
	3	02-004	.5	part
	4	01-001	1	lot
	5	05-007	5	part
	6	03-001	2	lot
	7	01-002	1	lot
	8	02-004	.325	part
	9	01-001	1	lot
	10	06-001	5	part
	11	03-001	4	lot
	12	01-001	1	lot
	13	02-003	.25	part
5	1	02-001	1	lot
	2	01-001	1	lot
	3	02-004	.611	part

Table 5.4: Raw material cost and quality data

Part number	Raw material cost per part	quality of raw material (%bad)
2	$1.05	0.5%
4	$ .35	5.0%
5	$ .19	1.0%

5.2.6 Other data

The function Fms that relates market share to the quality of product produced in previous years is given by,

$$Fms(p,n) = (((.8 \times BP(n-1,p) + .6 \times BP(n-2,p) + .3 \times BP(n-3,p) +$$

$$.1 \times BP(n-4,p)) - .10) + 1) * .3$$

The bad product fraction, (BP), for the last four years has been .05, .052, .054, .06. The product sells for $6.00. The before tax rate of return is 20%.

5.2.7 Results

$PESAW$ is calculated for the case where the current machine is retained. Tables 5.5 and 5.6 show intermediate results. $PESAW$ without replacement is $-517.86, with replacement is $735.63. This indicates that replacement is favored from an economic standpoint. It is interesting to note that this result is in spite of the fact that the $PEMRC$ of the new machine is greater than that of the old machine by almost 15%. The benefits do not stem from the cost of the equipment itself, but from the system effects.

Table 5.5: Sales, quality, material costs, and revenues

Yr	Company Sales		%Bad		Material Costs		Revenues	
	Def.	Ch.	Def.	Ch.	Def.	Ch.	Def.	Ch.
1	89,640	89,640	.060	.056	193,445	190,883	537,840	537,840
2	87,989	89,073	.066	.061	193,812	193,071	527,933	534,440
3	89.871	92,102	.072	.066	202,058	203,228	539,228	552,613
4	88,275	91,402	.077	.071	202,655	205,392	529,650	548,413
5	86,892	90,766	.083	.076	203,693	207,754	521,353	544,656

Table 5.6: *PEMRC* of processors with and without replacement

	PEMRC	with
Processor	Defender	Challenger
1	118,647	118,647
2	59,347	59,339
3	107,543	107,614
4	138,126	137,957
5	96,016	96,033
6	136,594	136,594
7	99,585	100,237
Def/Cha	163,205	187,917

Table 5.7: Potential *PESAW* with ideal quality characteristics

Processor changed	*PESAW*
none	- 517
1	174,434
2	33,642
3	31,276
4	4,600
5	7,704
6	92,510
7	214,846
8	296,533

5.3 Determination of Likely Replacement Candidates

The use of a systems model in replacement analysis can also be helpful to evaluate the current set of processors to determine which items of equipment are most likely candidates for replacement. As an example, using the data as given above, *PESAW* can be recalculated as each machine's quality characteristics are set to an ideal of $GB = 0$, $BG = 1.0$, $BD = 0$. (Other system models may suggest other criteria of evaluation.) The results of are shown in Table 5.7. The results indicate that the most potential benefit is to be gained from the replacement of processors 1,7, and 8.

5.4 Interactions of Replacement Alternatives

Given the data as already described, suppose that there is a competitor machine available to replace processor 7 with the characteristics given below in Table 5.8. There are now two possible replacements to be made. Table 5.9 shows the *PESAW*

Table 5.8: Characteristics of a challenger of processor 7

Function	*Challenger*
I	12,000
V	$\frac{75000}{n+1} + \frac{90000}{.0005u+1}$
PV	$9500 + V$
$aroc$	$28000 + 3000n$
$UROC$	$20u + (u/4)^2$
GB	.0
BG	.40
BD	.55
Operation	*time*
10 − 001	.0005

Table 5.9: $PESAW$ with various combinations of replacement

	Replace 8	Don't Replace 8
Replace 7	-517	735
Don't replace 7	722	-2089

values for the four combinations of replacement possible. Interestingly, while either replacement considered individually is profitable ($PESAW > 0$), if both are made the result is unacceptable. This shows that when using an integration view, the merit of replacing one item is dependent on the replacement of other items.

5.5 Computational Burden of Combinations of Replacement Decisions

As was shown in the third example, the decision for replacement of one item is not necessarily independent of decisions for replacement of others. This is true if the possible replacements occur in the same year or in different years. It is often not possible to determine the dependent/independent status of a replacement decision without analysis. The system modeling approach exemplified above provides a way to do such an analysis. However, when the assumption is made that all decisions cannot be treated as independent of all other decisions, the examination of a large number of replacement combinations is necessitated. For example, suppose in a manufacturing system there are M machines that could be replaced. Using a planning horizon of N years, let the number of possible alternatives for each machine m in year n (including keep the current machine) be denoted, $k(m,n)$. The total number of sets of decisions possible for the N year planning period is

$$\prod_{n=1}^{N}\left(\prod_{m=1}^{M} k(m,n)\right) \tag{5.19}$$

For a simple case with 10 machines, and a 5 year planning horizon, if $k(m,n) = 2$, for all m and n, there are 1.125 trillion possible decision sets. It would be impractical to compute the measure of merit for all these sets.

In Chapter 4, the technique utilized to reduce computation in replacement analysis was dynamic programming. Unfortunately, the dynamic programming approach is not applicable to the system model as it is formulated here. Consider the sample model cast as a sequential decision problem with decisions made at the beginning of each year to or not to replace each machine. The problem becomes one of moving from the beginning of year one to the end of the planning horizon with maximum $PESAW$. Since the costs of use of each processor depend on the age and on the cumulative utilization of the processor, the states in the decision network are constituted by a set of processors with given ages and cumulative utilizations along with a collection of bad product percentages for the previous years. All of these values will effect the value of any path to the next year. With the state so described, it is highly unlikely that two sets of decisions will ever lead to the same state. The problem becomes one of finding in a tree network the shortest path from the root to a leaf node. But for this problem dynamic programming is of no help.

An approach to reducing the number of paths to examine in the tree is through heuristics. It may be possible to find a probable optimum solution by looking at only a fraction of the possible paths. Preliminary studies by the author using the well known heuristic search algorithm described in Nilsson [1980], show some promise in this approach. With states in the decision tree defined as in the above paragraph, it is impossible to find the costs to take you to any state unless that state occurs in the last year of the planning horizon or if all machines are retired in that year. Otherwise the cost is dependent on how long the machines are retained, since the first cost is recovered over the duration of the equipment life. However, if one can sum up all operating costs and other known costs incurred up to the time of the state being examined and then makes a heuristic estimate of remaining costs an estimate can be made of the minimal cost path through any node in the decision tree. The past costs become Nilsson's $\hat{g}$ value and the heuristic estimate of minimum remaining costs becomes the $\hat{h}$ value.

Using this formulation and extending the data in Chapter 5 to include possible replacement machines for processors 7 and 8 in each of years 1 through 4, an optimal sequence of replacements was found by examining only 30 of the possible 256 sequences. Results of this analysis are found in Appendix C.

6 SUMMARY

The objective of this research was to extend current economic replacement models to make them more applicable to evaluating replacements where technological change and resulting indirect cost impacts are considered. An integrated approach was used that expanded the modeling boundaries to include the effects of changing demand and machine utilization and also to include the interactions of the various machines and other entities that make up a manufacturing system.

6.1 Use-based Operating Costs

First, the concept of utilization based operating costs and operating cost gradients was added to the traditional replacement model. Methodologies to find optimal replacements for various sets of assumptions were shown.

In the like-for-like replacement case, it was noted that the annual equivalent cost was not very sensitive to the utilization level, or even to the exact replacement timing. However, the cost per part produced was quite sensitive to the utilization, showing the effect of economies of scale.

If two machines are necessary to provide a single function, the costs can be minimized by running one machine at maximum capacity and the other machine at a level to meet the remaining demand. However, the savings may not be a large percentage since the costs are not highly sensitive to the mix of machines providing the service.

When the most general assumptions were considered, dynamic programming was used to reduce the burden of finding the optimal replacement sequence. For multiple machines providing the same function, a multi-dimensional dynamic programming algorithm could be used, given certain assumptions. Here, a two-dimensional algorithm was presented.

6.2 System Interactions

After describing the addition of utilization concepts to replacement analysis, a system model was presented to exemplify a modeling approach that included the effects of the interactions of system components. The major interaction accounted for in the sample model was that of part quality. Flaws created in the manufacturing process increased the load on the processors through rework. The amount of rework affected the utilization of the processors. Also the model incorporated the effect of bad product leaving the factory through a feedback effect on demand.

The system model, although only modeling one interaction, was very data intensive. It was useful for a comparison of alternative courses of action. However, to find an optimal course of replacements when all the processors in the system are considered requires the comparison of an exponentially large number of alternatives. Dynamic programming does not apply to this optimization process. A heuristic search procedure offers some promise. The system model also could be used as a tool to suggest likely candidates for replacement.

7 RECOMMENDATIONS FOR FURTHER STUDY

More research needs to be done to make these models practical for industrial decision makers. First, the important system interactions must be determined. In spite of what has been said in this article, there is a degree to which the machines in a manufacturing system act quite independently. It is therefore necessary to determine the ways in which one processor effects the costs and benefits of another significantly. Sensitivity analysis can be applied to various model formulations. Quality was an important factor in the model described here. Scheduling, material handling, and capacity can also be considered. It should be noted that the model developed by Leung and Tanchoco [1987] addresses these factors.

The "simplified" model described in Chapter 5 is data intensive. A complete systems economic model would use incredible amounts of data. For the model presented here, operation times and parts explosion data already exists in most manufacturing firms, often in computer readable form. Most larger firms have divisions to perform market research and estimate product demand. This same division might also estimate how demand changes as a function of quality. The data most challenging to obtain are the quality characteristics of machines and the degradation functions. Research will have to be done in procedures for determining such estimates.

Because of advancements in data collection devices the trend is to have more and more manufacturing information available to management. Already authors such as Knoop[1987] and Carrasco and Blank[1987] are basing economic analysis models on the availability of online manufacturing data. Still the issue of extrapolating historical data into the future remains.

Finally the heuristic approach to reducing evaluations of decision options needs more exploration. The heuristic, of course, depends on the model used. Perhaps some generalized approach to developing an heuristic can be developed. Testing to see how well such heuristics perform would also be necessary.

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10 APPENDIX A: SENSITIVITY ANALYSIS OF ECONOMIC LIFE

In the sensitivity analysis that examined the changes in economic life and present equivalent cost as the salvage value, use-related operating cost, and age-related operating cost changed, the following functions were used.

For salvage value:

$$\begin{aligned} 1 \quad V(N, cu(N)) &= 100 \mid N = 0 \\ &= 10 \mid N > 0 \end{aligned}$$

$$2 \quad V(N, cu(N)) = 100e^{-.1NU}$$

$$3 \quad V(N, cu(N)) = 100e^{.-05NU}$$

For *aroc*:

1 $aroc(N) = N$
2 $aroc(N) = 10N$ *linear*
3 $aroc(N) = 25N$

4 $aroc(N) = e^{.1N}$
5 $aroc(N) = e^{.15N}$ *convex*
6 $aroc(N) = e^{.3N}$

7 $aroc(N) = 100(1 - e^{.16N})$
8 $aroc(N) = 50(1 - e^{.16N})$ *concave*
9 $aroc(N) = 10(1 - e^{.16N})$

For $UROC$:

1 $UROC(N) = NU$
2 $UROC(N) = 10NU$ *linear*
3 $UROC(N) = 25NU$

4 $UROC(N) = e^{.1NU}$
5 $UROC(N) = e^{.15NU}$ *convex*
6 $UROC(N) = e^{.3NU}$

7 $UROC(N) = 100(1 - e^{.16NU})$
8 $UROC(N) = 50(1 - e^{.16NU})$ *concave*
9 $UROC(N) = 10(1 - e^{.16NU})$

The utilization, U, took on values of 100%, 75%, and 50%.

On the next three pages are samples of the outputs generated for the various combinations of the functions.

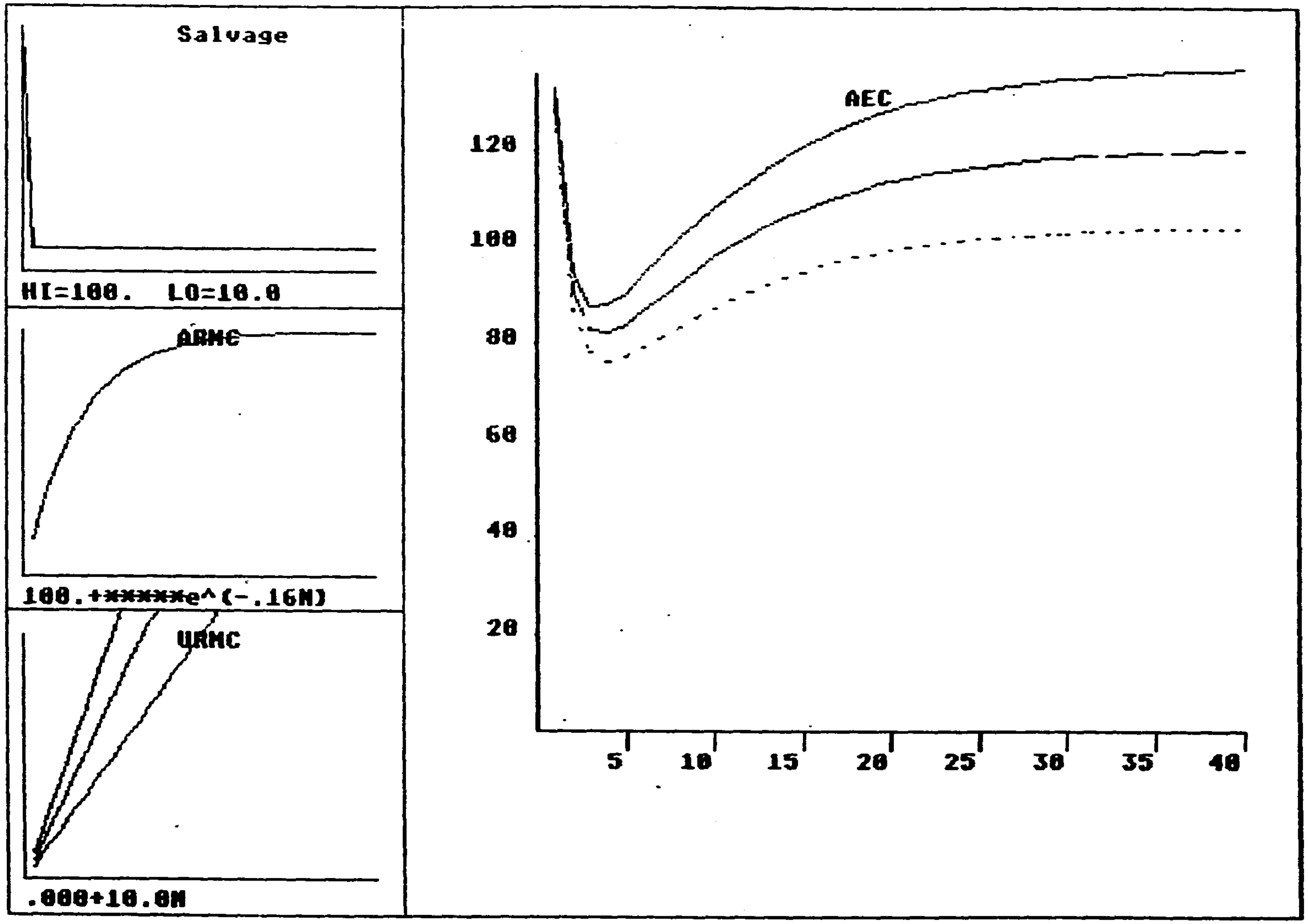

Figure 10.1: Sensitivity analysis: sample 1

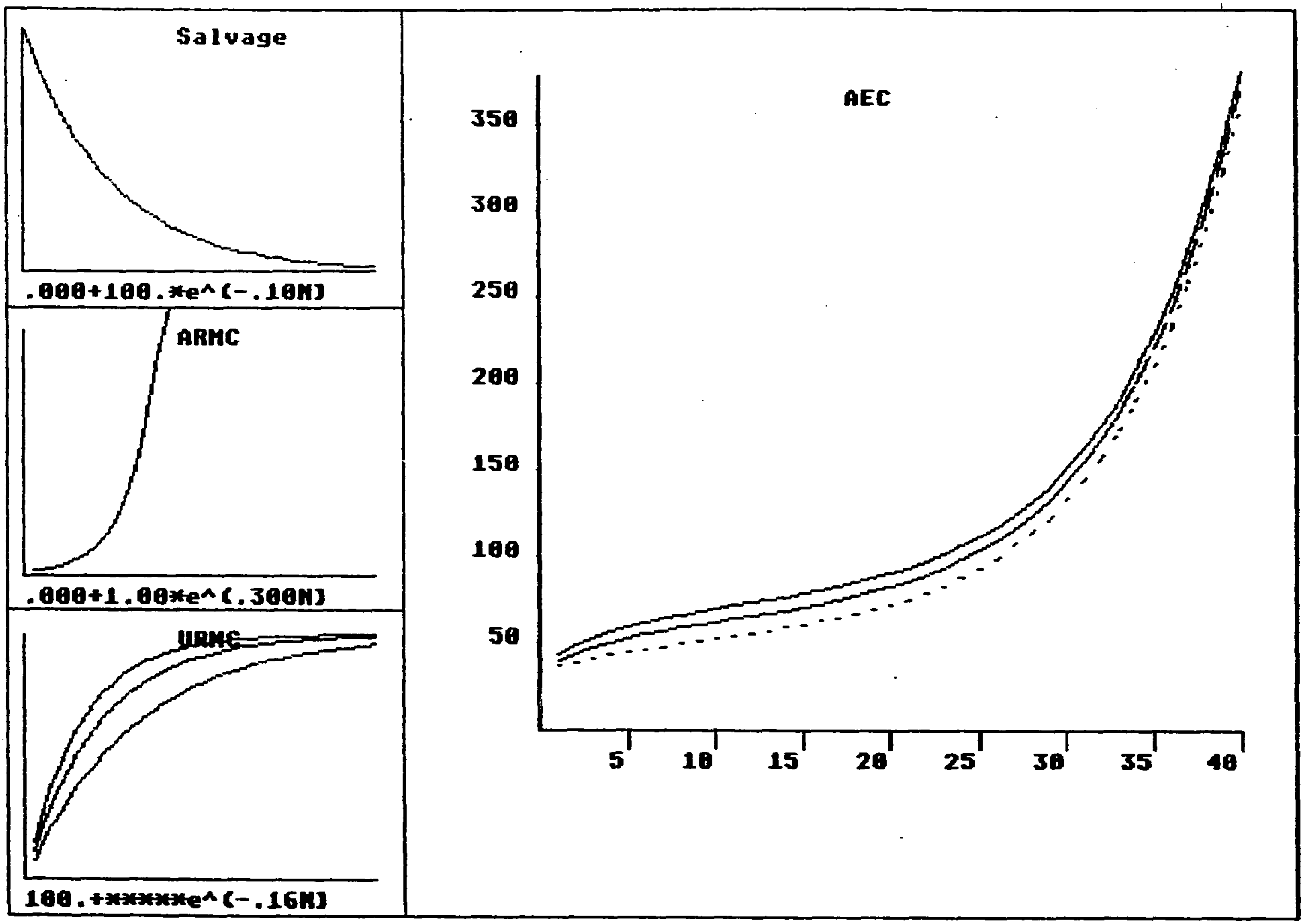

Figure 10.2: Sensitivity analysis: sample 2

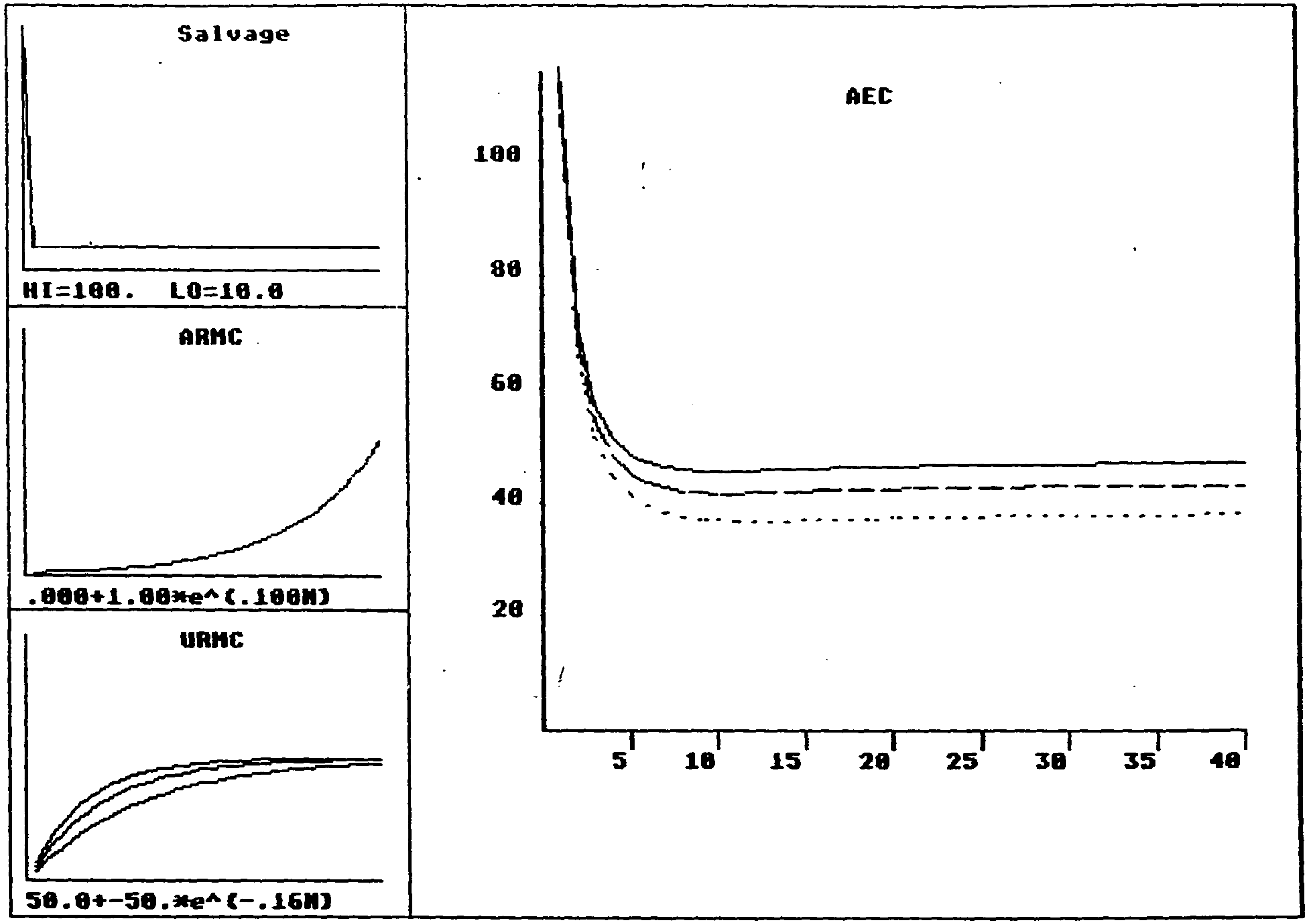

Figure 10.3: Sensitivity analysis: sample 3

11 APPENDIX B: DATA OF CURRENT PROCESSORS

1	I	=	0.0	*operations and times*
	$V(n,u)$	=	0.0	01 – 001 1.0
	$PV(n,u)$	=	$1000n + .75u$	01 – 002 1.5
	$aroc(n)$	=	$40000 + 2000n$	01 – 003 2.3
	$UROC(u)$	=	$.50u$	
	GB	=	$.01$	
	BG	=	$.35$	
	BD	=	$.25$	
2	I	=	0.0	*operation and time*
	$V(n,u)$	=	0.0	04 – 001 3.4
	$PV(n,u)$	=	$300n + .9u$	
	$aroc(n)$	=	$21000 + 300n$	
	$UROC(u)$	=	$2u$	
	GB	=	$.01$	
	BG	=	$.45$	
	BD	=	$.25$	

3 $I = 0.0$ *operation and time*

$V(n,u) = \frac{500}{(1+\frac{n}{10})} + \frac{15000}{(\frac{u}{2000}+1)}$ 09 − 015 .004

$PV(n,u) = 2500 + V(n,u)$

$aroc(n) = 29000 + 1700n$

$UROC(u) = 10(\frac{u}{75})^{1.5}$

$GB = .05 + .000001u$

$BG = .15$

$BD = .45$

4 $I = 0.0$ *operation and time*

$V(n,u) = \frac{7000}{n+1} + \frac{1250}{(1.+\frac{u}{2000})}$ 05 − 007 .002

$PV(n,u) = 3000 + V(n,u)$

$aroc(n) = 25000 + 1700n$

$UROC(u) = 15u + (\frac{u}{10})^{1.4}$

$GB = .0175 + .00000003u$

$BG = .15$

$BD = .10$

5 $I = 0.0$ *operation and time*

$V(n,u) = \frac{21000}{n+1} + \frac{9000}{\frac{u}{2000}+.5}$ 06 − 001 .0025

$PV(n,u) = 3000 + V(n,u)$

$aroc(n) = 15000 - 800n$

$UROC(n) = 12(\frac{u}{15})^{1.3}$

$GB = .025 + .00000007u$

$BG = .10$

$BD = .25$

6
$$\begin{aligned}
I &= 0.0\\
V(n,u) &= 121000\\
PV(n,u) &= 12000 + V(n,u)\\
aroc(n) &= 50000 + 1600n\\
UROC(u) &= 0\\
GB &= .01\\
BG &= .00\\
BD &= .00
\end{aligned}$$

operations and times

03 – 001 40.0

03 – 002 1.0

03 – 031 .0167

7
$$\begin{aligned}
I &= 0.0\\
V(n,u) &= 2000 * (20 - n) + \frac{10000}{(1 + \frac{u}{2000})}\\
PV(n,u) &= 1500 + V(n,u)\\
aroc(n) &= 23000 + 500n\\
UROC(u) &= .2u + (\frac{u}{390})^2\\
GB &= .02\\
BG &= .10\\
BD &= .30
\end{aligned}$$

operation and time

10 – 001 .0025

12 APPENDIX C: HEURISTIC SEARCH TO FIND OPTIMUM REPLACEMENT SEQUENCE

With a tree constructed as described in Chapter 5, the $\hat{g}$ value is computed by finding *PESAW* of the sequence of replacements leading to the given node. The $\hat{h}$ heuristic estimate used in the analysis was chosen to be an upper bound value. It was computed by ignoring capital costs, assuming that no defective parts would be produced, and calculating the processor operating costs for each operation using the lowest cost processor available in that year (including ones that would be purchased in a previous year).

There was one slight possibility that the $\hat{f}$ value would not be an upper bound. The $\hat{g}$ value used contained a salvage value received at the end of the time period. This value may not actually be received in some of the paths further down in the tree. So the estimate is not entirely an upper bound estimate, but almost.

Figure 12.1 is a partial search tree where the optimal value was found after examining only 30 scenarios of a possible 256. One machine has two challengers each year for the entire planning horizon of 5 years, and another machine has two challengers each year for the first three years only.

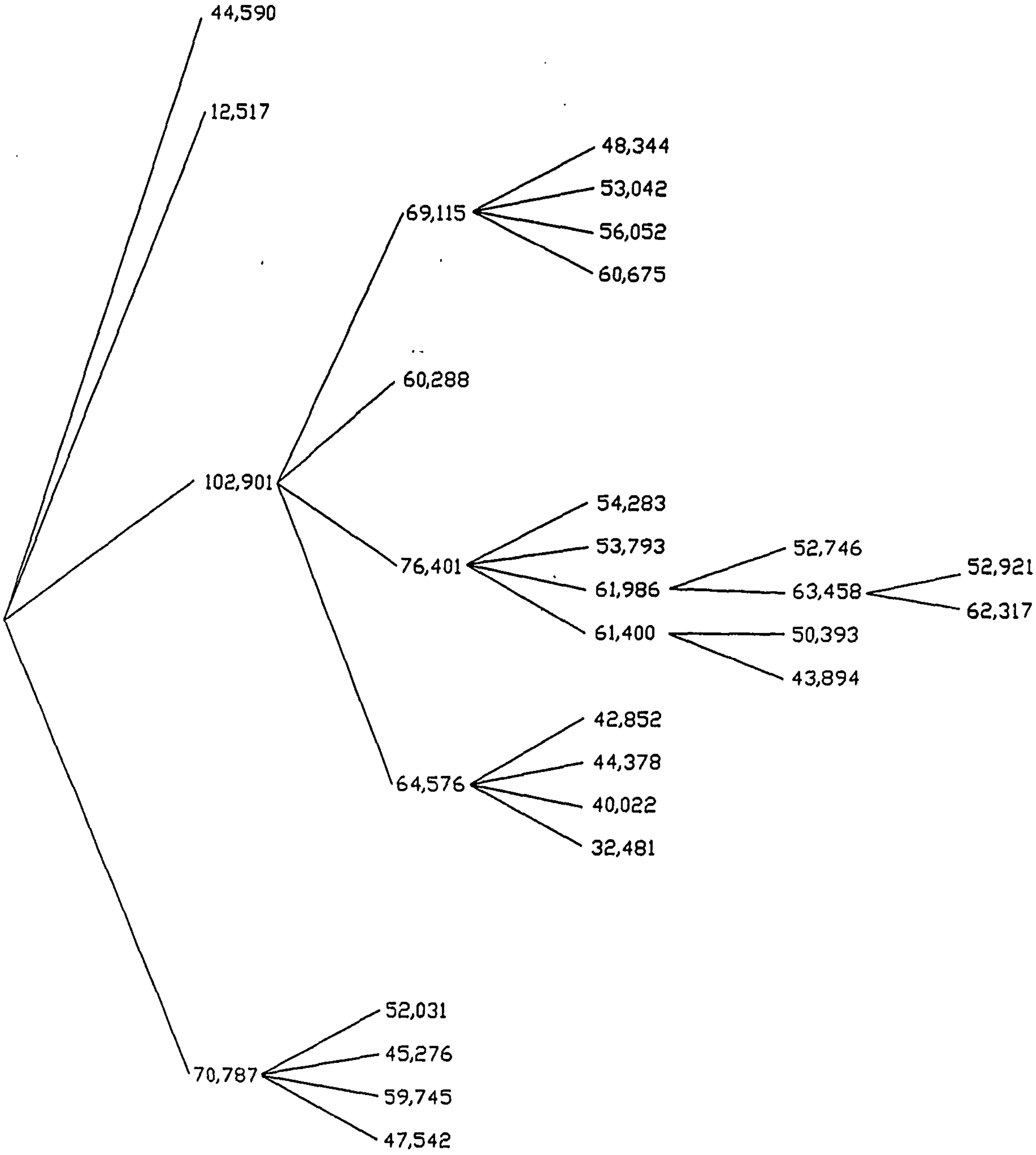

Figure 12.1: A search tree to find optimal replacements for 5 year planning horizon